**THE FACTOR THEOREM EXPLAINED.**

Consider the function ***f(x) = x2 – 6x + 8***



 *If x = 0 then f(0) = 0 – 0 + 8 = 8*

 *If x = 1 then f(1) = 12 – 6×1 + 8 = 3*

 ***If x = 2 then f(2) = 22 – 6×2 + 8 = 0 \*\*\****

 *If x = 3 then f(3) = 32 – 6×3 + 8 = – 1*

 ***If x = 4 then f(4) = 42 – 6×4 + 8 = 0 \*\*\****

 *If x = 5 then f(5) = 52 – 6×5 + 8 = 3*

Notice that ***f(x)*** can only be equal to 0 **twice**. ***f(2) = 0 and f(4) = 0***

We cannot find any other value which comes to zero because ***y = f(x)*** is the above parabola which can only cross the ***x*** axis up to 2 times.

If we factorise ***f(x)* we get *f(x) = (x – 2)(x – 4)***

It should now seem obvious that if we substitute ***x = 2*** we get ***f(2) = (0)×(-2)= 0***

and if we substitute ***x = 4,*** we get ***f(2) = (2)×(0)= 0***

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Now consider ***f(x) = x3 – 7x2 + 14x – 8 = (x – 1)(x – 2)(x – 4)***

then the only ***x*** values which make ***f(x) = 0*** are when ***x = 1 or 2 or 4.***

Another way of thinking of this is that if ***f(1) = 0 then (x – 1) must be a factor.***

**This is basically what we mean by the FACTOR THEOREM.**

**eg**

If ***f(x) = x3 – 7x2 + 14x – 8***

We TRY ***x = 1*** and get ***f(1) = 1 – 7 + 14 – 8 = 0 so (x – 1) is a factor***.

We TRY ***x = 2*** and get ***f(2) = 8 – 28 + 28 – 8 = 0 so (x – 2) is a factor***.

We TRY ***x = 3*** and get ***f(3) = 27 – 63 + 42 – 8 ≠ 0 so (x – 3) is NOT a factor***.

We TRY ***x = 4*** and get ***f(2) = 64 – 112 + 56 – 8 = 0 so (x – 4) is a factor***.

We KNOW that there are no more than 3 factors so:

***f(x) = x3 – 7x2 + 14x – 8 MUST BE EQUAL TO (x – 1)(x – 2)(x – 4)***

**Written in a GENERAL way:**

***If f(a) = 0 then (x –a) must be a factor of f(x)***

To find the factors of a function we do not have to keep trying different numbers to find when the expression becomes zero.

We only need 1 to start with.

1. Find the factors of ***f(x) = x3 – 9x2 + 26x – 24***

***Try x = 1, f(1) = 1 – 9 + 26 – 24 ≠ 0 so (x – 1) is not a factor!***

***Try x = 2, f(2) = 8 – 36 + 52 – 24 = 0 so (x – 2) IS A FACTOR***

***So we know f(x) = x3 – 9x2 + 26x – 24 = (x – 2)×(something)***

***There are two ways to find the “something”.***

***(i) We could do a long division:***

 ***x2 – 7x + 12***

 ***x – 2 x3 – 9x2 + 26x – 24***

 ***x3 – 2x2***

 ***– 7x2 + 26x***

 ***– 7x2 + 14x***

 ***12x – 24***

 ***12x – 24***

 ***0***

Then we factorise ***x2 – 7x + 12 = (x – 3)(x – 4)***

So ***f(x) = x3 – 9x2 + 26x – 24 = (x – 2)×(x – 3)(x – 4)***

***(ii) A very neat way to find the “something” is this:***

***We know f(x) = x3 – 9x2 + 26x – 24 = (x – 2)×(something)***

 ***= (x – 2) (x2 …*?*... + 12)***

***The x2 is obvious to get x × x2 = x3***  ***The 12 is also obvious which***

 ***comes from – 2×12 = – 24***

To get the “middle term” we find where it would come from….

***f(x) = x3 – 9x2 + 26x – 24 =***  ***(x – 2) (x2 …?... + 12)***

we already have ***– 2x2***

The other term in ***x2*** comes from ***x × – 7x***

***f(x) = x3 – 9x2 + 26x – 24 =***  ***(x – 2) (x2  – 7x. + 12)***

So now we have ***– 2x2*** and ***– 7x2* to make the *– 9x2* asrequired.**

This 2nd method needs some practice!

This would make a nice POSTER for frequent reinforcement of WHY.

