**ASSESSMENT SCHEDULE**

#### **91577 Apply the algebra of COMPLEX NUMBERS in solving problems**

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| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply the algebra of complex numbers in solving problems* involves:  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply the algebra of complex numbers, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing **limited knowledge of algebra of complex numbers in solving problems.**

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

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|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 1a |  | Correct expression. |  |  |
| 1b |  | Correct solution. |  |  |
| 1c | k – 1 = 2√x  (k – 1)2 = 4x    x = (k – 1)2/4 |  | Correct solution. |  |
| 1d | ***Let z rcisθ***  ***r3cis 3θ= k6cis(180 + 360n)***    ***r3 = k6 3θ=180 + 360n***  ***r = k2 θ = 60 + 120n***  ***z1 = k2 cis 600***  ***z2 = k2  cis 1800***  ***z3 = k2 cis 3000*** | Award u if changed correctly into polar form:  r3cis 3θ= k6cis(180+360n) | Correct solution either using degrees or radians. |  |
| 1e | ***If z = 3 + i then so is z = 3 – i***  ***Let the 3rd root (which is real) be p***  ***The equation could be written as:***  ***(z – p)( z2 – 6z + 10) = 0***  ***(using sum of roots = -(3 + i + 3 – i) = -6***  ***and product of roots = (3 + i)(3 – i) = 10)***  ***expanding and comparing:***  ***z3 – (p + 6)z2 + (10 + 6p)z – 10p = 0***  ***z3 + Az2 – 2z + B = 0***  ***the only one to use is the coeff of z:***  ***10 + 6p = -2 so p = -2***  ***The REAL solution is p = -2***  ***Equating coeff of z2: A = -(6 – 2) so A = -4***  ***Equating constant term: B = -10p = 20***    ***Summary: solutions 3 ±i and -2***  ***A = -4 and B = 20*** | Award u if  ***(z – p)( z2 – 6z+ 10)***  is obtained | Award r if student obtains A = -4 | Correct solution showing correct logical steps with correct mathematical statements required for t. |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 2a(i) | 11 + 10i | Correct expression. |  |  |
| 2a(ii) | ***u + v = 5 + 5i = 5√2(cis450) or***  ***= √50(cis450)*** | Correct or in rads |  |  |
| 2b | *If and  are the two roots then*    *x2 – bx + c = (x – α)( x – 4α)*  *= x – 5α + 4α2*  *b = 5α c = 4α2*  *α= b c = 4 b2*  *5 25* | Award u if  b = 5α c = 4α2 | Correct solution. |  |
| 2c | ***z = ( b – bi)×(1 – i)***  ***(1 + i) (1 – i)***  ***= (0 – 2bi) = – bi***  ***2***  ***Arg z = - π or - 900***  ***2*** | Award u for mult by ***(1 – i)***  ***(1 – i)***  correctly. | Accept 2700  Correct solution. |  |
| 2d | ***z – 4i = x + (y – 4)i × (x – 2) – yi***  ***z – 2 (x – 2) + yi (x – 2) – yi***  ***= x(x – 2) + y(y – 4) + i ( (x – 2)(y – 4)- xy***  ***(x – 2)2 + y2***  ***Real = 0 so x2 – 2x + y2 – 4y = 0***  ***Completing square***  ***x2 – 2x + 1 + y2 – 4x + 4 = 1 + 4***  ***(x – 1)2 + (y – 2)2 = 1 + 4 = 5*** | Correct line 1  for **u** | Correct line 3.  for **r** | Correct solution showing correct logical steps with correct mathematical statements.  Exclusion ***x≠2,y≠0*** not required for **t** |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 3a | ***(x – p)log3 = (x + p)log2***  ***xlog3 – plog3 = xlog2 + plog2***  ***x(log3 – log2) = plog2 + plog3***  ***x = plog2 + plog3***  ***(log3 – log2)*** | Correct expression. |  |  |
| 3b | ***x + 4 = t***  ***x***  ***x + 4 = xt***  ***4 = xt – x***  ***4 = x(t – 1)***  ***4 = x***  ***(t – 1)*** | Correct solution.  Restriction on t not required. |  |  |
| 3c | ***√2 + i√2 = 2cis450***  ***So ( 2cis45 )6 = 26 cis 2700***  ***=26(cos270 + isin270)***  ***= -26i or -64i*** | Correct polar form for u | Correct solution for r |  |
| 3d | ***r2cis2θ = 4cis(600 + 360n)***  ***r2 = 4 2θ = 60 + 360n***  ***r = 2 θ = 30 + 180n = 30, 210***  ***z1 = 2cis30 = 2(cos30 + isin30)***  ***= √3 + i***  ***z2 = 2cis210 =2(cos210 + isin210)***  ***= -√3 – i*** | Correct 1st line  for u | Correct solution.  for r |  |
| 3e | ***│ z +1 – i │ = │(1 – i) z│***  ***│ x + yi +1– i │ = │(1 – i) ( x + yi)│***  ***│(x +1) + (y – 1)i │ = │ (x + y) + ( y – x)i │***  ***(x +1)2 + (y – 1)2 = (x + y)2 + ( y – x)2***    ***x2+2x+1+y2–2y+1 = x2+2xy+y2+y2+x2–2xy***  ***2 = x2 – 2x + y2 + 2y***    ***2 + 1 + 1 = x2 – 2x + 1 + y2 + 2y + 1***  ***4 = (x – 1)2 + (y + 1)2*** | Correct 2nd line for u | Correct 4th line for r | Correct solution showing correct logical steps with correct mathematical statements. |

**ASSESSMENT SCHEDULE**

#### **91578 Apply DIFFERENTIATION methods in solving problems**

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| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply differentiation methods in solving problems* involves*:*  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply differentiation methods, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply differentiation methods, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question demonstrating limited knowledge of differentiation techniques

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 1a |  | Correct derivative. |  |  |
| 1b | . At *x* = 0  Equation of tangent sub (0, 2)  c= 2 so tan equ is | Correct derivative with correct solution. |  |  |
| 1c | =0.06366 | Correct chain for for u.  . | Correct derivative with correct solution. |  |
| 1d | ***(i)***  ***(ii)  does not exist***  ***(iii) is not differentiable at***    ***(iv) 1 < x < 3***  ***(v) x > 5*** | Correct answer for 2 situations for u | Correct answer for at least 3 situations for r |  |
| 1e | (x, y)  r y  x x  ***A = 2xy but x2 + y2 = r2***  ***So A = 2x (r2 – x2) ½***  ***dA = 2x ½ (r2 – x2) – ½(-2x)+2(r2 – x2) ½  = 0***  ***dx for max A***  ***2 (r2 – x2) ½  = 2x2***  ***(r2 – x2) ½***  ***r2 – x2 = x2***  ***r2 = 2x2***  ***x = r and y = r***  ***√2 √2***  ***Max A = 2xy = 2 r × r = r2***  ***√2 √2*** | Correct . | Correct maximum for *x*. | Correct solution. |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 2a |  | Correct derivative. |  |  |
| 2b | 2 4 | Correct graph of derivative crossing x axis at x = 0, 2 and 4. |  |  |
| 2c | At  Grad of tangent = | Correct . | Correct derivative with correct solution. |  |
| 2d |  | Correct . | Correct derivative with correct solution. |  |
| 2e | ***r 500***  ***800 – x x***  ***Total T = c(800 – x) + (5002 + x2) ½ 4c***  ***dT = c – 1 + ½ 4×(2x) = 0 for min***  ***dx (5002 + x2) ½***  ***4x = 1***  ***(5002 + x2) ½***  ***4x = (5002 + x2) ½***  ***16x2 = 5002 + x2***  ***15x2 = 5002***  ***x2 = 5002  x ≈ 129 m***  ***15*** | Correct equ for cost = u | Correct derivative for r | Correct derivatives with correct solution. |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 3a |  | Correct derivative. |  |  |
| 3b | ***yꞌ = ½ ex/2 = ½ when x = 0***  ***grad of normal = - 2***  ***if x = 0, y = 1***  ***equ of normal is y = -2x + 1*** | Correct derivative with correct solution. |  |  |
| 3c | ***4y3 yꞌ + yꞌ = 3x2 – 1***  ***yꞌ(4y3 + 1) = 3x2 – 1***  ***yꞌ = 3x2 – 1 = 11***  ***4y3 + 1 5*** | Correct 1st line for u | Correct solution. |  |
| 3d | ***ie***  ***and***      ***At h = 8  [ = 0.159]*** | Correct | Correct derivative with correct solution. |  |
| 3d | ***y = exx2 + ex***  ***yꞌ= ex2x + exx2 + ex***  ***= ex(x2 + 2x + 1)***  ***= ex(x + 1)2 = 0 at max/min/infl***  ***So x = -1 since ex never equals zero.***  ***1st derive test for x = -1***   |  |  |  |  | | --- | --- | --- | --- | | ***x*** | ***-2*** | ***-1*** | ***0*** | | ***yꞌ*** | ***+*** | ***0*** | ***+*** |   ***Stationary infl pt at x = -1, y = 2e – 1***  ***yꞌꞌ = ex2 + ex2x + ex2x + exx2  + ex***  ***= ex(2 + 2 + 2x + x2 + 1)***  ***= ex(x2 + 4x + 3)***  ***= ex (x + 1)(x + 3) = 0 at infl pts***  ***1st derive test for x = -3, y = 10e – 3***   |  |  |  |  | | --- | --- | --- | --- | | ***x*** | ***-4*** | ***-3*** | ***-2*** | | ***yꞌ*** | ***+*** | ***0*** | ***+*** |   ***y***  ***-3 -1 x***  ***Curve is concave down for -3 < x < -1*** | Correct ***yꞌ for u*** | *Correct* ***yꞌꞌ for r*** | Correct derivatives with correct solution. |

**ASSESSMENT SCHEDULE**

#### **91579 Apply integration methods in solving problems**

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| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply integration methods in solving problems* involves*:*  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply integration methods, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply integration methods, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing limited knowledge of integration techniques.

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  | | | **Not Achieved** | **Achievement** | | | **Achievement with Merit** | | | **Achievement with Excellence** | | |
| **Score range** | | | 0 - 6 | 7 - 13 | | | 14 - 18 | | | 19 -24 | | |
| Question ONE | | | Evidence | | | Achievement (u) | | | Merit (r) | | | Excellence (t) | | |
|  | | |  | | | *Apply integration methods in solving problems.* | | | *Apply integration methods, using relational thinking, in solving problems.* | | | *Apply integration methods, using extended abstract thinking, in solving problems.* | | |
| 1a(i) | | | ***6e2x + c or 3e2x  + c***  ***2*** | | | Correct integration. | | |  | | |  | | |
| 1a(ii) | | | dx = *3x4 + ln│x│ + c* | | | Correct integration.  not essential. | | |  | | |  | | |
| 1b | | | ***u = x – 2 so du = dx and x = u + 2***  dx  =  = | | | Correct substitution line 2 for u | | | Correct integration with correct solution. | | |  | | |
| 1c | | | ***dv = a***  ***dt***  ***so v = at + c subs v = u, t = 0 so c = u***  ***v = dx = u + at***  ***dt***  ***x = ut + ½ at2 + d subs x = 0, t = 0***  ***so d = 0***  ***hence x = ut + ½ at2*** | | | 2 Correct integrations. | | | Correct  Integrations with both constants c and d evaluated. | | |  | | |
| 1d | | | Intersection points: x(x – 1)(x – 3) = x(x – 3)  B  A  x(x – 1)(x – 3) – x(x – 3) = 0  x(x – 3) ( x – 1 – 1) = 0  x(x – 3)(x – 2) = 0  intersection points x = 0, 2 and 3  Area A = cubic – parabola  = – 4x2 + 3x – (x2 – 3x) dx  = x4 – 4x3 + 3x2 – (x3 – 3x2)  4 3 2 3 2  = 2.6667 by calc or 8  3  Area B = parabola – cubic    = (x2 – 3x) – (x3 – 4x2 + 3x) dx  = (x3 – 3x2) – ( x4 – 4x3 + 3x2 )  3 2 4 3 2  = 0.4165 by calc or 5  12    Total area between curves = 3.083 or 37  12 | | | Correct intersection for u | | | Correct idea of area = area under cubic – area under parabola (and vice versa) shown with equations. | | | Correct solution showing correct integrations and correct mathematical logic and statements.  If middle intersection is only “assumed” to be  x = 2 only award an ***r*** | | |
| Question TWO | | Evidence | | | | | Achievement (u) | | | Merit (r) | | | Excellence (t) | |
|  | |  | | | | | *Apply integration methods in solving problems.* | | | *Apply integration methods, using relational thinking, in solving problems.* | | | *Apply integration methods, using extended abstract thinking, in solving problems.* | |
| 2a | | ***sin 3x + cos 5x + c***  ***3 5*** | | | | | Correct integration. | | |  | | |  | |
| 2b | | A =  = ***31k5*** | | | | | Correct integration.  (accept 32k5 – k5) | | |  | | |  | |
| 2c | | = | | | | | Correct integration with minor error in division (or substitution).  not essential. | | | Correct integration with correct solution. | | |  | |
| 2d | | Area  ***63a = 315***  ***a = 5*** | | | | | Correct integration. | | | Correct integration with correct solution. | | |  | |
| 2e | | ***dv = 9.8e – 0.175t***  ***dt***  ***v = - 9.8 e – 0.175t + c***  ***0.175***  ***Subs t = 0, v = 0 so 0 = -56 + c***  ***v = 56 – 56e – 0.175t***  ***as t increases e – 0.175t 0***  ***so terminal velocity is 56 m/s*** | | | | | Correct integral  for u | | | Correct equation  v = 56 – 56e – 0.175t  for r | | | Correct solution showing correct integrations and correct terminal vel with clear mathematical statements. | |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.* |
| 3a(i) | .= 3sin4x + c | Correct integration. |  |  |
| 3a(ii) | ***tan 4x – cot 2x + c***  ***4 2*** | Correct integration. |  |  |
| 3b | ***y =ex – ln │x│ + c***  ***subs x = 1, y = 2***  ***2 = e – 0 + c so c = 2 – e***  ***y = ex – ln│ x│ + 2 – e*** | Correct integration. | ││ not essential  Correct integration with correct constants |  |
| 3c | ***Large area = e4 – 1***  ***Small area = ek – 1***  ***2(ek – 1) = e4 – 1***  ***2ek – 2 = e4 – 1***  ***2ek = e4 + 1***  ***ek = e4 + 1 = 27.799***  ***2***  ***k = 3.325*** | 2 Correct definite  integrations. | Correct integrations with correct calculation of k. |  |
| 3d | ***x = sint so dx = cost dt***  ***√(1 – x2) = √(1 – sin2t) = cost***  ***If x = 1 , t = π***  ***2***  =  ***Use cos 2t = 2 cos2t – 1***  ***So cos2t = ½ ( cos 2t + 1)***  π/2  = ½ sin 2t + t  2 0  = 1 × π = π  2 2 4 | Reducing integral to cos2t for u | Using cos 2t formula for r | Correct integration with correct logical steps and correct solution. |