**SUMMARY OF DIFFERENTIATION TECHNIQUES.**

***You need to master the basic techniques used in differentiating functions.***

***THE CHAIN RULE.***

***dy = dy × dt***

***dx dt dx***

***This is used over and over again. It may be useful to remember it in “words”.***

***eg y = ( x2 + 3x + 5)7***

***y = ( something )7***

***so dy = 7 × ( the same thing )6 × (the derivative of the something)***

***dx***

***ie dy = 7 ( x2 + 3x + 5)6 × (2x + 3)***

***dx***

***THE PRODUCT RULE.***

***If y = u × v***

***then dy = u × dv + du × v or dy = u × dv + v × du as on formula***

***dx dx dx dx dx dx sheet***

***In words : dy = 1st × derivative of 2nd + derivative of 1st × second***

***dx***

***eg 1. if y = 8x3 (x + 4)6***

***dy = 8x3 × 6(x + 4)5 + 24x2 × (x + 4)6***

***dx***

***2. (combining product rule and chain rule)***

***If y = (5x + 4)7 × (x2 – 3x + 6)9***

***Then using dy = 1st × derivative of 2nd + derivative of 1st × second***

***dx***

***we get dy = (5x + 4)7 × 9(x2 – 3x + 6)8(2x – 3) + 7(5x + 4)6×5 × (x2 – 3x + 6)9***

***dx***

***THE QUOTIENT RULE.***

***If y = u***

***v***

***then dy = v × du – u × dv***

***dx dx dx***

***v2***

***In words : dy = bottom × derivative of top – top × derivative of bottom***

***dx (bottom)2***

***eg 1 If y = x3 then dy = (5x + 2) × 3x2 – x3 × 5***

***(5x + 2) dx (5x + 2)2***

***2. If y = x2 + 6x + 7 then dy = (x2 – 4x – 3) ×(2x + 6) – (x2 + 6x + 7) ×(2x – 4)***

***x2 – 4x – 3 dx (x2 – 4x – 3)2***

***3. (combining quotient rule and chain rule)***

***If y = (x3 + 4x)6 then dy = (x2 – 3x)5 × 6(x3 + 4x)5(3x2+ 4) – (x3+ 4x)6 ×5(x2 – 3x)4(2x – 3)***

***(x2 – 3x)5 dx (x2 – 3x)10***

***4. Often we need to simplify, when finding max/min values.***

***IF y = x2 – 3 FIND WHERE THE GRADIENT IS ZERO.***

***x – 2***

***y ′ = (x – 2) ×2x – (x2 – 3) ×1 = 0***

***(x – 2)2***

***So 2x2 – 4x – x2 + 3 = 0***

***x2 – 4x + 3 = 0 ie (x – 1)(x – 3) = 0***

***So gradient is zero when x = 1 and when x = 3***

***EXPONENTIAL FUNCTIONS. If y = ex then y ′ = ex***

***Chain Rule :***

***Using “words” y = e (something) so y ′ = e ( same thing) × (derivative of the something)***

***1. If y = e (5x + 6) then y ′ = e (5x + 6) × 5***

**(x3 + 4x + 2)**

**(x3 + 4x + 2)**

***2. If y = e then y ′ = e× (3x2 + 4)***

***Product Rule : In words y ′ = 1st × derivative of 2nd + derivative of 1st × second***

***eg. If y = e4x (x2 + 8)6 then y ′ = e4x × 6(x2 + 8)5×2x + 4e4x × (x2 + 8)6***

***Quotient Rule :***

***In words : y ′ = bottom × derivative of top – top × derivative of bottom***

***(bottom)2***

***eg If y = e3x + 5 then y ′ = (e6x – 4) × 3e3x – (e3x + 5) × 6e6x***

***e6x – 4 (e6x – 4)2***

***TRIGONOMETRICAL FUNCTIONS.***

***If y = sin x then y ′ = cos x and if y = cos x then y ′ = – sin x***

***Chain Rule :***

***Using “words” y = sin(something) then y ′ = cos(same thing) × derivative of the something***

***eg 1. If y = sin ( x2 + 3x + 5) then y ′ = cos (x2 + 3x + 5) × (2x + 3)***

***2. If y = sin (ex) then y ′ = cos (ex) × ex***

***3. If y = e sin x then y ′ = e sin x × cos x***

***4. If y = sec x = 1 = (cos x) – 1  show that y ′ = sec x tan x***

***cos x***

***5. If y = cosec x = 1 = (sin x) – 1  show that y ′ = –cosec x cot x***

***sin x***

***Product Rule : In words y ′ = 1st × derivative of 2nd + derivative of 1st × second***

***eg If y = e5x × sin x then y ′ = e5x × cos x + 5e5x × sin x***

***Quotient Rule :***

***In words : y ′ = bottom × derivative of top – top × derivative of bottom***

***(bottom)2***

***eg.1 If y = e3x  then y ′ = sin x × 3e3x  – e3x × cos x***

***sin x sin 2x***

***2. If y = tan x = sin x use the quotient rule to show y ′ = sec2 x***

***cos x***

***3. If y = cot x = cos x use the quotient rule to show y ′ = – cosec2 x***

***sin x***

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***(A) If y = sin (x4) (B) If y = (sin x)4 or sin 4 x***

***then y ′ = cos (x4) × 4x3 then y ′ = 4 (sin x)3 × cos x***

***Special Note 2: A triple chain***

***If y = sin* 7*(4x) = sin 4x* 7 *then y ′ = 7 sin 4x 6 × cos4x × 4***

***Explained in full : If y = sin* 7*(4x) = sin (4x)* 7**

***Let y = t* 7  *where t = sin u and u = 4x***

***So dy = 7t6 and dt = cos u and du = 4***

***dt du dx***

***so dy = dy × dt × du = 7 sin6(4x) × cos(4x) × 4***

***dx dt du dx***

***LOGARITHMIC FUNCTIONS. If y = ln(x) then y ′ = 1***

***x***

*(****Note: ln (x) means loge (x)***

***( however on scientific calculators log x means log10 x )***

***Chain Rule :***

***Using “words” y = log(something) so y ′ = 1 × (derivative of the something)***

***( same thing)***

***1. If y = ln(x2 + 3x + 4) then y ′ = 1 × (2x + 3) or (2x + 3)***

***(x2 + 3x + 4) (x2 + 3x + 4)***

***2. If y = ln(7x) then y ′ = 1 × 7 = 1***

***7x x***

***3. If y = ln( sin x) then y ′ = cos x = cot x***

***sinx***

***4\*. If y = ln (ex) then y ′ = 1 × ex = 1 so loge(ex) must be x***

***ex***

***5\*. If y = ln (2x + 3)5 it is better to use the “log laws” and change it to :***

***y = 5 ln (2x + 3) so y ′ = 5 × 1 × 2***

***2x + 3***

***6\*. If y = ln 4x – 2 it is also better to use the “log laws” and change it to :***

***x2 + 1***

***y = ln (4x – 2) – log (x2 + 1) so y ′ = 4 – 2x***

***4x – 2 x2 + 1***

***7. If y = tan x then y ′ = ln x × sec2x – tan x × 1***

***ln x x***

***( ln x )2***