**IMPLICIT DIFFERENTIATION EXPLAINED.**

Normally, when we differentiate an expression we simply write the following:

***If y = 5x3***

then ***dy = 15x2***

***dx***

We need to re-think ***dy***

***dx***

in a different way:

we think of ***d*** ( ) as meaning :

***dx***

“Differentiate the *x*’s in the brackets”

Sometimes this is referred to as :

“Differentiate ***with respect to*** *x*” but it is perhaps more instructive to think of it as “**Differentiate the *x*’s”**

So ***d ( x3 + 6x2 + 2x)*** means :

***dx***

“ **Differentiate the *x*’s** ” and of course it equals ***3x2 + 12x + 2***

Similarly,  ***d ( y4 + 7y)*** means :

***dy***

“ **Differentiate the *y*’s** ” and this equals ***4y3 + 7.***

Also,  ***d (t2 + 5t + 3)*** means

***dt***

“ **Differentiate the *t*’s** ” and this

equals ***2t + 5.***

Now consider the expression :

***d ( t5 )***

***dx***

This means to **differentiate the *x*’s but there are no *x*’s, only *t*’s.**

Here we use a version of the chain rule normally stated as:

***dy = dy × dt***

***dx dt dx***

we must think of this as :

***d ( t5 ) = d ( t5 ) × dt***

***dx dt dx***

so ***d ( t5 ) = 5t4 × dt***

***dx dx***

**The DIFFERENCE between EXPLICIT and IMPLICIT equations.**

Firstly an **Ex**plicit equation has ***y*** as the subject and just ***x***’s on the other side.

Eg ***y = x3 + x2 – 4x + 7***

An **Im**plicit equation has ***x***’s and ***y***’s mixed throughout the equation.

Eg ***y3 + x4 + 5y2 – 7x = 2***

It is often not possible to transform an implicit equation into an explicit equation so we need this method to differentiate such equations.

Consider the equation :

***y3 + x4 + 5y2 – 7x = 2***

If possible, it is quite a good idea to have ***x***’s on one side of the equation and ***y***’s on the other even though we cannot make ***y*** the single subject.

***y3 + 5y2 = 2 + 7x – x4***

Now we differentiate both sides of the equation *with respect to* ***x.***

***d ( y3 + 5y2 ) = d( 2 + 7x – x4)***

***dx dx***

Remember the symbol ***d ( )***

***dx***

means, differentiate the ***x***’s in the brackets.

We can do the right hand side but the left hand side has ***y***’s **not** ***x***’s.

So using the chain rule idea explained earlier, we do the following :

***dy × d ( y3 + 5y2 ) = d( 2 + 7x – x4)***

***dx dy dx***

***dy ( 3y2 + 10y) = 7 – 4x3***

***dx***

***dy = (7 – 4x3)***

***dx (3y2 + 10y)***

Normally, we would set out the answer as follows:

Find ***dy if e3y + sin y = ln(x) + x4***

***dx***

***3e3y dy + cos y dy = 1 + 4x3***

***dx dx x***

***dy ( 3e3y + cos y) = 1 + 4x3***

***dx x***

***dy = 1 + 4x3***

***dx x***

***(3e3y + cos y)***

**EXTENSION.**

Sometimes we need to use the product rule or quotient rule as well:

Eg Find ***y ′*** if :

***x3y5 + xy = 9x***

***x3 5y4 y ′ + 3x2y5 + xy ′ + 1 y = 9***

***y ′ (x3 5y4 + x ) = 9 – 3x2y5 – y***

***y ′ = (9 – 3x2y5 – y)***

***( x3 5y4 + x )***

A particularly difficult point is finding ***d2y*** for Parametric equations.

***dx2***

***eg y = t3 + t2 x = ln(t)***

***dy = 3t2 + 2t dx = 1***

***dt dt t***

***dy = dy × dt***

***dx dt dx***

***= (3t2 + 2t) × t***

***= 3t3 + 2t2***

***d2y = d dy***

***dx2 dx dx***

***d2y = d 3t3 + 2t2***

***dx2 dx***

***d2y = dt × d 3t3 + 2t2***

***dx2 dx dt***

***= t × (9t2 + 4t)***

***= 9t3 + 4t2***

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Finally, an interesting point is that we can find ***y ′*′** in the following case in **three** ways : **parametrically, implicitly and explicitly.**

**PARAMETRICALLY:**

***x = sin t y = cos t***

***dx = cos t dy = – cos t***

***dt dt***

***dy = dy × dt = – sin t = – tan t***

***dx dt dx cos t***

***d2y = – d ( tan t) = – dt × d ( tan t)***

***dx2 dx dx dt***

***= – 1 × sec2 t = – 1 = – 1***

***cos t cos3t y3***

**IMPLICITLY:**

Consider ***x2 + y2 = 1***

***So 2x + 2y y′ = 0***

***y′ = – x***

***y***

***and y′′ = – y × 1 – x × y′***

***y2***

***= – y + x2***

***y***

***y2***

***= – y2 + x2 = – 1***

***y3  y3***

***EXPLICITLY:***

***y = ( 1 – x2 ) ½  (ignoring ±)***

***so y′ = ½ ( 1 – x2 ) – ½ × (– 2x)***

***= – x***

***(1 – x2) ½***

***And y′′ = – (1 – x2) ½ – x2 (1 – x2) – ½***

***(1 – x2)***

***= – (1 – x2) ½ – x2***

***(1 – x2) ½***

***( 1 – x2)***

***= – (1 – x2) – x2***

***(1 – x2) 3/2***

***= – 1***

***y3***