**ASSESSMENT SCHEDULE**

#### **91577 Apply the algebra of COMPLEX NUMBERS in solving problems**

|  |  |  |
| --- | --- | --- |
| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply the algebra of complex numbers in solving problems* involves:  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply the algebra of complex numbers, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing **limited knowledge of algebra of complex numbers in solving problems.**

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 1a | ***f(1) = 0***  ***so 3c – 4c – 8c +18 = 0***  ***18 = 9c***  ***2 = c*** | Correct expression. |  |  |
| 1b | ***u = 1 + i = 2 ½ cis(450)***  ***u8 = 24cis3600*** | Correct solution.  Degrees or rads |  |  |
| 1c | ***4x – 8 = x2 – 4x + 4***  ***0 = x2 – 8x + 12***  ***0 = (x – 2)(x – 6)***  ***x = 2 or 6***  ***check if x = 2 check if x = 6***  ***lhs = 0, rhs = 0 lhs = 4 rhs = 4***  ***so x = 2 is valid so x = 4 is valid*** | Award u if minor error  ( or no check done) | Award r only for  Correct solution with check. |  |
| 1d | ***Let z rcisθ* z*5 = 32ni***    ***r5cis 5θ= 32ncis(90 + 360n)***    ***r5 = 32n 5θ=90 + 360n***  ***r = 2n0.2 θ = 18 + 72n***  ***z1 = 2n0.2 cis 180***  ***z2 = 2n0.2  cis 900***  ***z3 = 2n0.2 cis 1620***  ***z4 = 2n0.2cis2340***  ***z4 = 2n0.2cis3060*** | Award u if changed correctly into polar form:  r5cis 5θ=  ***32ncis(90 + 360n)*** either using degrees or radians. | Correct solution either using degrees or radians. |  |
| 1e | =  ***│(x+1) + (y + 1)i│=│(x + 2) + iy│***  ***Squaring each modulus:***  ***(x + 1)2 + (y + 1)2 = (x + 2)2 + y2***  ***x2 + 2x + 1 + y2 + 2y + 1 = x2 + 4y + 4 + y2***  ***2x + 2 + 2y = 4y + 4***  ***2x – 2 = 2y***  ***y = x – 1*** | Award u if  ***│(x+1) + (y + 1)i│***  ***=│(x + 2) + iy│***  is found correctly | Award r if  ***(x + 1)2 + (y + 1)2 = (x + 2)2 + y2***  is obtained | Correct solution showing correct logical steps with correct mathematical statements. |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 2a | *z2 – 2z + 25 = 0*  *z2 – 2z + 1 = 1 – 25*  *(z – 1)2 = -4×6*  *z – 1 = ±i2√6*  *z = 1 ± i2√6* | Correct expression. |  |  |
| 2b | ***u = 8 – 4i***  ***v = 1 – 3i***  ***w = 2 + 2i correctly placed at (2, 2)*** | If ***w*** correctly worked out and placed on Argand diagram award u |  |  |
| 2c | ***1 root of v3 – 2v2 – 3v + k = 0*** is ***–1 + 2i***  ***another is –1 – 2i (conjugate)***  ***Let 3rd root (which must be real) = b***  ***(z – b)(z2 + 2z + 5) = 0***  ***Comparing terms in z2: 2 – b = -2***  ***So b = 4 = 3rd solution***  ***Comparing number terms: -5b = k***  ***-20 = k*** | Award u if the  root ***–1*** ***– 2i*** is stated | Correct solution. |  |
| 2d | ***(x – 2)2 + (y + 2)2 = 4***  ***(Locus is a circle, centre (2, -2) radius 2)***    Min value of **Im(z) = -4** | Award u for correct locus drawn. | Award r if min value also stated. |  |
| 2e | ***z = (a + bi) (3 – i) = 3a + b + i(3b – a)***  ***(3 + i) (3 – i) 10 10***  ***3b – a = -1 if angle = -450***  ***3a + b 3b-a***  ***-45***  ***So 3b – a = - 3a – b 3a-b***  ***4b = -2a so a = -2b*** | Correct expression without ***i2***  for u | Arg(z) = - 450 or -1350  interpreted to give a  correct relationship such as  im(z) = -1 for r  re(z) | Correct solution. |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 3a | (1 + √2)(3 – √2)2 =(1 + √2)(11 - 6√2)  = -1 + 5√2 | Correct answer required for u |  |  |
| 3b | ***u = 8cis(5θ)*** and ***v = 4cis(3θ)***  ***8cis (5θ) = 2cis(2θ)***  ***4cis(3θ)*** | Correct solution.  required for u |  |  |
| 3c | )9  1  = ( 1cis(-45) )9  = 1 cis(-405) = cis (-45)  = ) | Change to polar form required for u | Correct solution  required for r |  |
| 3d | ***w = 1*  *= 1 × (x – iy)2***  ***z2 (x + iy)2 (x – iy)2***  ***= x2 + y2 – 2xyi***  ***(x + iy)(x – iy)(x + iy)(x + iy)***  ***= x2 + y2 – 2xyi = x2 + y2 – 2xyi***  ***(x2 + y2) (x2 + y2) (x2 + y2)2*** | Multiplied by conjugate in some form for u  Could have expanded 1 . .  (x + iy)2  at start but method shown is better. | Correct **exact** solution for r. |  |
| 3e | ***(i) x4 + px3 + qx2 + rx + t = 0***  ***Equ is (x – a)(x – b)(x – c)(x – d) = 0***  ***(x2 – (a+b)x + ab)(x2 – (c+d)x + cd)***  ***Coeff of x3: p = – (a + b) – (c + d)***  ***So a + b + c + d = –p***  ***And...***  ***(x2 – (a+b)x + ab)(x2 – (c+d)x + cd)***  ***Number term: t = abcd***  ***(ii)***  ***Dividing: –p = a + b + c + d***  ***t abcd***  ***= a + b + c + d .***  ***abcd abcd abcd abcd***  ***= 1 + 1 + 1 + 1***  ***bcd acd abd abc*** | Correct 2nd line for u | Compared coefficient of ***x3*** to get 1st relationship  and number term to get 2nd. | Divided correctly and clearly to get part (ii) |

**ASSESSMENT SCHEDULE**

#### **91578 Apply DIFFERENTIATION methods in solving problems**

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| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply differentiation methods in solving problems* involves*:*  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply differentiation methods, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply differentiation methods, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question demonstrating limited knowledge of differentiation techniques

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) |  |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 1a | ***y = 8sin(4x)***  ***yꞌ = 32cos(4x)*** | Correct derivative. |  |  |
| 1b | ***y = (x3 + 2x)2***  ***yꞌ = 2(x3 + 2x)(3x2+2) = 2×3×5 = 30***  ***so gradient of normal = -1***  ***30*** | Correct answer = u |  |  |
| 1c | ***x = cos(2t)*** and ***y = 2sin(t)***  ***dx = –2sin(2t) dy = 2cos(t)***  ***dt dt***  ***dy = 2cos(t) = cos(t) = – 1 .***  ***dx –2sin(2t) –cos(t)sin(t) sin(t)*** | Correct up to  ***dy = 2cos(t)***  ***dx -2sin(2t)***  = u | Correctly simplified = r |  |
| 1d | ***y = e 8 – 4x  + 4x***  ***yꞌ= -4e8 – 4x + 4 = 0 at stat pts***  ***so 4 = 4e8 – 4x***  ***1 = e8 – 4x***  ***0 = 8 – 4x***  ***So x = 2*** | ***Correct dy for u***  ***dx***  ***and state that***  ***dx = 0***  ***dt*** | Correct answer for for r |  |
| 1e | If the slant height of a cone is 8√3 cm prove that the maximum volume  is when ***h*** = 8 cm and ***r*** = 8√2 cm      ***r2+ h2 = 192***  ***r2= 192 – h2***  ***h*** 8√3  ***r***  ***V = π (192 – h2)h = π (192h – h3)***  ***3 3***  ***dV = π (192 – 3h2) = 0 for max vol***  ***dh 3***  ***192 = 3h2  so h2 = 64***  ***h = 8 (not -8)***  and ***r2= 192 – h2***  ***= 192 – 64***  ***= 128***  ***= 64×2***  ***So r = 8√2*** | Correct Equ for V by eliminating r or h but preferably r | Correct derivative and solving for derivative = 0 | Correct r and h  found for t |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 2a | ***y = ln(x2 + 1)***  ***3x + 4***  ***yꞌ =(3x + 4)× 2x – ln(x2 + 1) ×3***  ***(x2 + 1) .***  ***(3x + 4)2*** | Correct derivative. |  |  |
| 2b | ***y = 3e2x – 8***  where ***x = 4***  ***yꞌ = 6e2x – 8***  ***sub x = 4***  ***yꞌ = 6e0 = 6×1 = 6*** | Correct derivative and sub ***x = 4*** to get 6  for u |  |  |
| 2c | ***(i) not cont if x = 0, 1***  ***(ii) not diff if x = 0, 1, 4***  ***(iii)not defined if x = 0***  ***(iv) lim f(x) = 1***  ***x⭢ 0***  ***(v) lim f(x) = not defined***  ***x⭢ 1***  ***(vi) f(1) = 3*** | 3 Correct for u | 5 Correct for r |  |
| 2d | ***V = 16 – t – 16 (t + 1) – 1***  ***dV = - 1 + 16(t + 1) – 2  = 0 for max V***  ***dt***  ***16 = 1 so (t + 1)2 = 16***  ***(t+1)2***  ***t + 1 = ±4 so t = 3 or -5 but 0* ≤ *t* ≤ 15**  ***subs t = 3 max V = 16 – 3 – 4 = 9cm3*** |  |  |  |
| 2e | ***dx = 2cm/sec Area = base×height***  ***dt = (10 – x)× x2***  ***10***  ***A = x2 – x3***  ***10***  ***dA = 2x – 3x2***  ***dx 10***  ***dA = dA × dx = (2x – 3x2 ) × 2***  ***dt dx dt 10***  ***sub x = 5 , dA = (10 – 7.5) ×2 = 5 cm2/sec***  ***dt*** | Correct equation for area of rectangle for u  A = ***= (10 – x)× x2***  ***10*** | Correct chain of derivatives for r | Correct derivatives with correct solution for t |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 3a | ***y = (x3 + 2)–4 /5***  ***yꞌ = – 4 (x3 + 2) -9/5 × 3x2***  ***5*** | Correct derivative. |  |  |
| 3b | ***y = x2***  ***x – 2***  ***yꞌ = (x – 2)2x – x2 = 0 for stat pts***  ***(x – 2)2***  ***So 2x2 – 4x – x2 = 0***  ***x2 – 4x = 0***  ***x(x – 4) = 0***  ***x = 0 or 4*** | Correct derivative with correct solutions. |  |  |
| 3c | ***y = xln(x) – 2x***  ***yꞌ = x× 1 + lnx – 2***  ***x***  ***= 1 + lnx – 2***  ***= lnx – 1***  ***Decreasing if yꞌ < 0***  ***So lnx – 1 < 0***  ***lnx < 1***  ***x < e1*** | Correct derivative  for u | ***x < e***  ***is needed for r*** |  |

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| 3d | ***M***  ***h***  ***θ***  ***Q 800 C***  ***dh = - 90 h = tan θ***  ***dt 800***  ***dh = 800sec2 θ = 800***  ***dθ cos2 θ***  ***d θ = d θ × dh***  ***dt dh d t***  ***d θ = cos2 θ ×(–90)***  ***dt 800***  ***If h = 500 then tan θ = 5 so θ = 320***  ***8***  ***d θ = cos2 θ ×–90 = cos232 ×–90***  ***dt 800 800***  ***d θ = –0.0809 rad/sec***  ***dt*** | ***dh = - 90***  ***dt***  ***h = tan θ***  ***800***  ***and***  ***dh = 800sec2 θ dθ for u*** | all correct for r  accept decreasing at 0.0809 rad/sec |  |
| 3e | ***y***  A  S R  ***y***    ***x 600***    **B P O Q C *x***  OC = 10 OA = 10√3 from ∆OAC  ***Choose x,y axes as above.***  ***Equ of AC is y = – √3 x + 10√3***  ***AreaPQRS = 2x(– √3 x + 10√3)***  ***Area = – 2√3 x2 + 20√3 x***  ***Area = 2√3 (–x2 + 10x)***  ***d(Area) =2√3( – 2x + 10) = 0 for max***  ***dx***  ***so x = 5, PQ = 10 and QR = 10√3 – 5√3***  ***= 5√3***  ***max Area = 50√3*** | Use a coordinate system to approach the problem, use angles are 600 and calculate OA for u | Calculate correct equ for OA  for r | Correct derivatives with correct solution.  Finding max area  for t |

**ASSESSMENT SCHEDULE**

#### **91579 Apply integration methods in solving problems**

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| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply integration methods in solving problems* involves*:*  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply integration methods, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply integration methods, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing limited knowledge of integration techniques.

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | | Evidence | Achievement (u) | | Merit (r) | | Excellence (t) | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | *Apply integration methods in solving problems.* | | *Apply integration methods, using relational thinking, in solving problems.* | | *Apply integration methods, using extended abstract thinking, in solving problems.* | |
| 1a | | ***= –4x – 3  – ln(x) + c***  ***3*** | Correct integration. | |  | |  | |
| 1b | | =  =  = = 2 | Correct integration. | |  | |  | |
| 1c | | ***v = 2t –***  Find the distance moved from ***t*** = 1 to ***t*** = 4  ***x = t2 + e 1 – t  + c***  ***sub t = 4 x4 = 16 + e – 3  + c***  ***sub t = 1 x1 = 1 + e0 + c = 2 + c***  ***x4 – x1 = 14 + e – 3  ≈14.05 metres*** | Antiderivative correct. | | Correct integration with correct solution. | |  | |
| 1d | | ***dm =***  ***dt***  ***t*** = 0 ***m = 8g*** and at ***t = 2*** hr, ***m = 1 g.***  =  = ***kt + c***  ***= kt + c***  ***Sub t = 0, m = 8***  ***6 = c***  ***Sub t = 2, m = 1***  ***3 = 2k + 6 so k = – 9***  ***2 4***  ***= + 6***  ***If m = 0 t = 24 = hours***  ***9*** | Correct diff equ with k and c found  = u | | Correct time = r | |  | |
| 1e | | The graphs have the equations  ***y = (p2+1) – x2***and ***y = p2 where p > 1***  ***x2***  ***Intersection when (p2***+ 1) ***– x2 = p2***  ***x2***  ***(p2 + 1)x2 – x4 = p2***  ***0 = x4 – (p2 + 1)x2 + p2***  ***(x2 – 1)(x2 – p2) = 0***  ***x = ±1 and x = ± p***  ***area =***  ***=***  ***–***  ***= 2p3 – 2p2 + 2p – 2***  ***3 3*** | Correct intersection for u | | Correct idea of area = area under cubic – area under parabola (and vice versa) shown with equations for r. | | Correct solution showing correct integrations and correct mathematical logic and statements. | |
|  | |  |  | |  | |  | |
| Question TWO | Evidence | | | Achievement (u) | | Merit (r) | | Excellence (t) |
|  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | *x* | 2 | 4 | 6 | 8 | 10 | 12 | 14 | | *f(x)* | 3 | 9 | 13 | 15 | 11 | 5 | 1 | |  | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |   *A =2 3+ 1 + 4(9 + 15 +5) + 2(13+11)*  *3*  *= 112* | | | *Apply integration methods in solving problems.* | | *Apply integration methods, using relational thinking, in solving problems.* | | *Apply integration methods, using extended abstract thinking, in solving problems.* |
| 2a | Correct calculation. | |  | |  |
| 2b | ***y* ꞌ = *4x3 + 3x2 – 4x + c sub x = 1, y* ꞌ = 4**  ***4 = 4 + 3 – 4 + c so c = 1***  ***y = x4 + x3 – 2x2 + x + d sub x = 1, y = 2***  ***2 = 1 + 1 – 2 + 1 + d so d = 1***  ***y = x4 + x3 – 2x2 + x + 1*** | | | Correct integrations and constants evaluated and solution found = u. | |  | |  |
| 2c | A + B = ***dx = 8ln(8)***  So A = 4ln(8)  A = dx = 8ln(k)  Solving 8ln(k) = 4 ln(8)  ln(k) = ½ ln(8) = ln( 8  ½)  k = 8 ½ | | | Correct value for total area for u. | | Correct integration with correct value for k. | |  |
| 2d | ***x =* 1**when ***y =* 1**, find ***y*** when ***x = 2***  = =  ***ln(y) = – 3 + c***  ***x2***  sub ***x =* 1**and ***y =* 1**  ***0 = –3 + c so c = 3***  ***ln(y) = – 3 + 3***  ***x2***  sub ***x = 2***  ***ln(y) = – ¾ + 3 = 2 ¼***  ***y = e 2.25  ≈ 9.49*** | | | Correct integration with correct constant. | | Correct integration with correct constant and ***y*** value found. | |  |

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| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| 2e | =  =  = ***let u = ex + 1***  ***du = exdx***  ***=***  ***= ln(u) + c***  ***= ln( ex + 1) + c*** | - | Correct manipulations of  needed to integrate = r | Correct solution  = t |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.* |
| 3a | **= *dx***  ***= x3 + 4x + 8ln*|*x*| *+ c***  ***3*** | Correct integration.  **|*x*|** not essential |  |  |
| 3b | **= =**  = | Correct integration. |  |  |
| 3c | ***x – 3 .***  ***x + 3 x2 + 0x + 4***  ***x2 + 3x***  ***– 3x + 4***  ***– 3x – 9***  ***+ 13***  =  ***= x2 – 3x + 13 ln(x + 3) + c***  ***2*** | Correct division or similar = u | Correct  Integration evaluated = r. |  |
| 3d | ***I*** =  ***Let u = x + 2 so x = u – 2 and x – 2 = u – 4***  ***du = dx***  ***I = equ* A**  =  = – 8  = 4 | Uses substitution to obtain equ A | Evaluates integral correctly |  |
| 3e | ***dR = kR***  ***dt***  ***ln(R) = kt + c***  ***sub t = 100, R = 150 :***  ***ln(150) = 100k + c equ1***  ***sub t = 200, R = 90 :***  ***ln(90) = 200k + c equ2***  ***subtracting :***  ***ln(90) – ln(150) = 100k***  ***so k = -0.005108***  ***sub in equ 1***  ***c = 5.5214***  ***sub in the equ: ln(R) = kt + c***  ***ln(R) = –0.005108t + 5.5214***  ***if R = 20 : ln(20) = –0.005108t + 5.5214***  ***so t ≈ 494 days*** | Obtains ***dR = kR***  ***dt***  and states ***equ 1*** and ***equ 2*** for grade u | Obtains ***dR = kR***  ***dt***  and calculates ***c***  and ***k*** for grade r | Obtains  ***t ≈ 490 days*** with all correct mathematical statements for grade t. |