**AN UNUSUAL PROOF ON 2014 COMPLEX NUMBER ASSESSMENT.**

If ***u = x + iy*** and ***au2 + bu + c = 0***, prove that ***a u 2 + b u + c = 0***

I do remember teaching this type of problem perhaps over 30 years ago.

The “model” answer gave the following:



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However a most elegant “proof” is as follows:

If ***au2 + bu + c = 0 + 0i***

***then the conjugate of both sides must be true:***

 ***au2 + bu + c = 0 + 0i***

 ***au2 + bu + c = 0 – 0i***

 ***a ( u 2) + b ( u ) + c = 0 – 0i***

 ***a( u ) 2 + b ( u ) + c = 0 – 0i***

The only slight problem being that I stated that the conjugate of ( ***u2*** )

is the conjugate of ***u*** **then** squared!

ie ( ***u2*** ) = ( ***u*  )2**

To verify this, I suppose we should compare:

***(x + iy)2 = (x2 – y2) + 2xyi = (x2 – y2) – 2xyi***

and ***( x + iy )2 = ( x – iy )2 = (x2 – y2) – 2xyi***

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Alternatively, surely we could have proceeded as follows:

The solution of ***au2 + bu + c = 0***  by the quadratic formula is:

***u = – b ± √ (b2 – 4ac)***

 ***2a***

Since we are talking about **complex roots** we assume ***b2 – 4ac*** is **negative**

so let ***b2 – 4ac = – p2***

then ***u*** = ***– b ± √ (– p2 )***

 ***2a***

 = ***– b ± pi***

 ***2a***

 ie both roots ***– b + pi and – b – pi*** will satisfy the equation.

 ***2a 2a***

These roots are obviously complex conjugates equivalent to

***u = x + iy and u = x – iy***

Hence if ***u = x + iy*** and ***au2 + bu + c = 0***, then ***a u 2 + b u + c = 0***