**“PHANTOM GRAPH THEORY” APPLIED TO THE FUNDAMENTAL THEOREM OF ALGEBRA.**

The fundamental theorem of Algebra basically states that a polynomial equation of the form *axn + bxn – 1  + ….. + px2 + qx + r = 0* has *n* solutions.

**Only the highest power of *x* is significant.**

So *x****2*** *+ 5x + 4 = 0* has 2 solutions

 *x****3*** *+ 4x2 + 7x + 2 = 0* has 3 solutions

 *x****7*** *+ x6 + …… + 4x + 3 = 0* has 7 solutions.

Clearly some of these solutions are REAL and some are COMPLEX numbers.

eg *x2 – 2x + 2 = 0*

 *x2 – 2x = – 2*

 *x2 – 2x + 1 = 1 – 2*

 *(x – 1)2 = – 1*

*x – 1 = ± i*

 *x = 1 ± i* Notice these 2 complex solutions are CONJUGATES

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**The fundamental theorem of Algebra would be better stated as: Any horizontal plane will cross a polynomial function, generally written as: *y = axn + bxn – 1  + ….. + px2 + qx + r* a total of “*n*” times, so that any equation *y = “a constant”* has “*n”* solutions.**

See below: We can see any horizontal plane (corresponding to any real *y* value)

will cut any parabolic function exactly two times.



**PART 2**

Considering a curve whose highest power is *x4*, we see that:

Line 1 does cross the curve 4 times

Line 2 SEEMS to cross the curve just 2 times

Line 3 SEEMS to not cross the curve at all.

 2

 1

 3

But when we consider the fact that “Phantom Curves” appear at each turning point and at right angles to the basic curve, we see that any horizontal plane (corresponding to any real *y* value) will cut such a curve 4 times, so that any equation ***y = “a constant”*** always does have 4 solutions.



 If this curve is *y = (x + 1)2(x – 1)2 = x4 – 2x2 + 1*

The horizontal plane *y = 9* will cross this curve 4 times

Solving *x4 – 2x2 + 1 = 9*

we get : *x4 – 2x2 – 8 = 0*

so *(x + 2)(x – 2)(x2 + 2) = 0*

giving *x = ±2* and *±√2* ***i***

 Notice there are 2 real solutions *x = 2* and *x = –2*

and 2 complex solutions *0 + 2i* and *0 – 2i* (which are CONJUGATES)

**PART 3**

Considering a curve whose highest power is *x3*, we see that:

Line 1 does cross the curve 3 times

Line 2 SEEMS to cross the curve 1 time

Line 3 SEEMS to cross the curve 1 time.

 2

 1

 3

But when we consider the fact that “Phantom Curves” appear at each turning point and at right angles to the basic curve, we see that any horizontal plane (corresponding to any real *y* value) will cut such a curve 3 times, so that any equation ***y = “a constant”*** has 3 solutions.



If this curve is *y = x(x – 3)2 = x3 – 6x2 + 9x*

Suppose we want to find *x* values when *y = 20*

We solve *x3 – 6x2 + 9x = 20*

Rearranging *x3 – 6x2 + 9x – 20 = 0*

Factorising *(x – 5)(x2 – x + 4) = 0*
producing the solutions *x = 5* or *x = 1 ±√(1 – 16) = 1 ± i√15
 2 2 2*Also notice that the complex solutions are always CONJUGATES.

Suppose we want to find *x* values when *y = – 50*

We solve *x3 – 6x2 + 9x = – 50*

Rearranging *x3 – 6x2 + 9x + 50 = 0*

 *(x + 2)(x2 – 8x + 25) = 0
 x = – 2 or 4 ± 3i*

Also notice that the complex solutions are always CONJUGATES.

In fact it should now be obvious that the complex solutions always occur in CONJUGATE PAIRS as long as the polynomial equation has real coefficients.

(this means that in equations like : *ax3 + bx2 + cx + d = 0*

 the values of a, b, c and d are real numbers.)

(N.B. The solutions do not occur in conjugate pairs if there are complex coefficients in the equation eq *x3 = 3 + 4i*)
If we restrict ourselves to cubic equations, there are really only 2 cases:

1. The equation has 3 REAL solutions

eg *x3 – 6x2 + 11x – 6 = 0*

 *(x – 1)(x2 – 5x + 6) = 0*

 *(x – 1 )( x – 2)(x – 3) = 0*

 *x = 1, 2 or 3*

1. The equation has 1 REAL solution and 2 COMPLEX CONJUGATE solutions.

eg *x3 – 5x2 + 17x – 13 = 0*

 *(x – 1)(x2 – 4x + 13) = 0*

 *x = 1 or x = 4 ± √(16 – 52)*

 *2*

 *x = 1 or x = 4 ± √( – 36)*

 *2*

 *x = 1 or x = 2 ± 3i*

**An equation cannot have just 1 complex solution because they always come in conjugate pairs**.

Incidentally if 2 of the roots of *x4 +ax3 + cx2 + d = 0* are *2 + 4i* and *5 – 3i* then quite simply, we can say that the other two solutions MUST be *2 – 4i* and *5 + 3i* because the solutions always come in conjugate pairs.

In this case, the original equation would have been :

*(x – (2 + 4i))(x – (2 – 4i))(x – (5 + 3i))(x – (5 – 3i)) = 0*

 *(x2 – 4x + 20)(x2 – 10x + 34) = 0*

 *x4 – 14x3 + 94x2 – 336x + 680 = 0*

SEE FULL VERSION : [www.phantomgraphs.weebly.com](http://www.phantomgraphs.weebly.com)