PROBLEMS ON FACTOR AND REMAINDER THEOREMS.

1. *Show that* $(x - 2)$ *is a factor of* $f(x) = x^3 - 9x^2 + 26x - 24$

2. *If* $(x - 2)$ *is a factor of* $f(x) = x^3 - 6x^2 + cx - 8$ *find c and factorise f(x) fully.*

3. *If* $(x - 4)$ *is a factor of* $f(x) = x^3 + bx^2 + 26x - 24$ *find b and factorise f(x) fully.*

4. *If* $(x + 1)$ *is a factor of* $f(x) = x^3 + px^2 - x + 6$ *find p and factorise f(x) fully.*

5. *If* $(x - 4)$ *is a factor of* $f(x) = x^3 - 4x^2 + ax + 16$ *find a and factorise f(x) fully.*

6. *If* $(x - 3)$ *is a factor of* $f(x) = x^3 - 3x^2 - 9x + c$ *find c and factorise f(x) fully.*

7. $(x - 3)$ *and* $(x - 4)$ *are both factors of* $x^3 + ax^2 + bx - 24$ *. Find a and b.*

8*. *If* $(x - \sqrt{3})$ *is a factor of* $f(x) = x^3 + bx^2 - 3x + 3$ *find b and factorise f(x) fully.*

9. $f(x) = x^3 + x^2 + x + 2$ *.*
If f(x) is divided by $(x - 1)$ *find the remainder R.*

10. *If the remainder R is 5 when* $f(x) = x^3 + bx^2 + 3x + 2$ *is divided by* $(x - 1)$ *find b.*

11. *If* $f(x) = x^2 + bx + c$ *is divided by* $(x - 1)$ *the remainder is 7*
If f(x) is divided by $(x - 2)$*, the remainder is 12. Find b and c.*

ANSWERS

1. Show that $(x-2)$ is a factor of $f(x) = x^3 - 9x^2 + 26x - 24$

By Factor Th^m if $(x-2)$ is a factor then
$f(2) = 0$
Check : $f(2) = 8 - 36 + 52 - 24 = 0$
So $(x-2)$ is a factor.

2. If $(x-2)$ is a factor of $f(x) = x^3 - 6x^2 + cx - 8$ find c and factorise f(x) fully.

By Factor Th^m if $(x-2)$ is a factor then
$f(2) = 0$
so $f(2) = 8 - 24 + 2c - 8 = 0$
$2c = 24$
$c = 12$
$f(x) = (x-2)(x^2 - 4x + 4) = (x-2)^3$

3. If $(x-4)$ is a factor of $f(x) = x^3 + bx^2 + 26x - 24$ find b and factorise f(x) fully.

By Factor Th^m if $(x-4)$ is a factor then
$f(4) = 0 = 64 + 16b + 104 - 24$
$16b = -144$ so $b = -9$
$f(x) = (x-4)(x^2 - 5x + 6)$
$= (x-4)(x-2)(x-3)$

4. If $(x+1)$ is a factor of $f(x) = x^3 + px^2 - x + 6$ find p and factorise f(x) fully.

By Factor Th^m if $(x+1)$ is a factor then
$f(-1) = 0 = -1 + p + 1 + 6$
so $p = -6$
$f(x) = (x+1)(x^2 - 7x + 6)$
$= (x+1)(x-6)(x-1)$

5. If $(x-4)$ is a factor of $f(x) = x^3 - 4x^2 + ax + 16$ find a and factorise f(x) fully.

By Factor Th^m if $(x-4)$ is a factor then
$f(2) = 0 = 64 - 64 + 4a + 16$
$4a = -16$ $a = -4$
$f(x) = (x-4)(x^2 - 4)$
$= (x-4)(x+2)(x-2)$

6. If $(x-3)$ is a factor of $f(x) = x^3 - 3x^2 - 9x + c$ find c and factorise f(x) fully.

By Factor Th^m if $(x-3)$ is a factor then
$f(3) = 0 = 27 - 27 - 27 + c$
$c = 27$
$f(x) = (x-3)(x^2 - 9)$
$= (x-3)(x-3)(x+3)$

7. $(x-3)$ and $(x-4)$ are both factors of $x^3 + ax^2 + bx - 24$. Find a and b.

By Factor Th^m if $(x-3)$ and $(x-4)$ are factors then
$f(3) = 0 = 27 + 9a + 3b - 24$
$9a + 3b = -3$
So $3a + b = -1$
And $f(4) = 0 = 64 + 16a + 4b - 24$
$16a + 4b = -40$
$4a + b = -10$
Solving $a = -9, b = 26$

8*. If $(x - \sqrt{3})$ is a factor of $f(x) = x^3 + bx^2 - 3x + 3$ find b and factorise f(x) fully.

By Factor Th^m if $(x - \sqrt{3})$ is a factor then
$f(\sqrt{3}) = 0 = 3\sqrt{3} + 3b - 3\sqrt{3} + 3$
$3b = -3$ so $b = -1$
$f(x) = (x - \sqrt{3})(x^2 + \sqrt{3}x - x - \sqrt{3})$
$= (x - \sqrt{3})(x + \sqrt{3})(x - 1)$

OR

If $(x - \sqrt{3})$ is a factor then so is $(x + \sqrt{3})$
$(x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3$
So $f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 1)$

9. $f(x) = x^3 + x^2 + x + 2$. If f(x) is divided by $(x - 1)$ find the remainder R.

By remainder Th^m $R = f(1)$
$R = 1 + 1 + 1 + 2 = 5$

10. If the remainder R is 5 when $f(x) = x^3 + bx^2 + 3x + 2$ is divided by $(x - 1)$ find b.

By remainder Th^m $R = f(1)$
$5 = 1 + b + 3 + 2$
$b = -1$

11. If $f(x) = x^2 + bx + c$ is divided by $(x - 1)$ the remainder is 7
If f(x) is divided by $(x - 2)$, the remainder is 12. Find b and c.

By remainder Th^m $R = f(1) = 7$
So $7 = 1 + b + c$
$b + c = 6$
By remainder Th^m $R = f(2) = 12$
So $12 = 4 + 2b + c$
$2b + c = 8$
Giving $b = 2$ and $c = 4$