**COLLECTION OF RELATED RATE PROBLEMS.**

1. A

 A ladder AC, 10m long, is leaning against a vertical wall AB.

 *y* 10 m ladder

 B *x* C

The top of the ladder is sliding down the wall at 0.3m/s.

Find at what rate the bottom of the ladder is moving when *x* = 8 m

 **y**

 A

 ***Notice, as ladder slips, A is moving in negative y direction***

 ***but C is moving in positive x direction.***

 ***B C x***

 ***x2 + y2 = 100***

***dy = – 0.3 so 2x + 2y dy = 0***

***dt dx***

 ***dy = – x = – 8***

 ***dx y 6***

***dx = dx dy = – 6 × – 0.3 = 0.225m/s***

***dt dy dt 8***

2. The volume of liquid in a container is given by *V = 2 π h3*

 *3*

where h is the depth in cm at time *t* seconds.

This container is being filled with a chemical at the rate of 8 cm3 per second.

Find the rate of increase of the depth of the liquid when the depth is 2 cm.

***dV = 8 dV = 2 π h2***

***dt dh***

***dh = dh × dV***

***dt dV dt***

***= 8 = 1 cm/sec***

 ***8 π π***

 ***= 1 × 8***

 ***2 π h2***

**3.**  The volume, V of a spherical bubble is : V = 4 π r3

 3

If the Volume is increasing at a rate of 12 cm3/ sec find the rate of increase of the radius when r = 1 cm.

***dV = 4 π r2  dr = dr × dV = 1 × 12 = 3***

***dt dt dV dr 4 π r2  π***

4. The volume of a spherical weather balloon, as it rises through the atmosphere, is increasing at a rate of 2000 cm3/min.

Find the rate of increase of the radius at the point when the radius r = 10 cm.

***We want dr***

 ***dt***

***Given dV = 2000 and V = 4πr3***

 ***dt 3***

 ***so dV = 4πr2***

 ***dr***

 ***Chain rule : dr = dr × dV = 2000 = 1.59 cm/min***

 ***dt dV dt 4πr2***

5.A plane is flying **horizontally** at 200 Km/h at a height of 3 Km.

 Find the rate at which the distance ***z***, from the plane P to an observer

 at O, is increasing when the horizontal distance ***x***, to the plane is 8 Km

 ✈ P

***z*** Km

 3 Km

 O ***x*** Km Q

***z2 = x2 + 32 so z = (x2 + 9)  ½***

**OR  *z2 = x2 + 9***

 ***2z dz = 2x***

 ***dx***

 ***dz = x***

 ***dx z***

***dz = dz × dx***

***dt dx dt***

 ***= x × 200***

 ***z***

 ***= 8× 200 = 187 Km/h***

 ***8.544***

***dz = dz × dx***

***dt dx dt***

 ***= x × 200***

 ***(x2 + 9)  ½***

 ***= 8 × 200***

 ***(82 + 9)  ½***

 ***= 187 Km/h***

6. A stuntman at S, is being filmed by a camera at C, as he slides down a vertical

 metal pole at a constant rate given by *dh = – 2 m/s*

 *dt*

S Find the rate at which the camera is

 rotating ( *dθ* ) when the stuntman

 h *dt*

 has descended to a height of 5 m

 θ above the ground.

 20 m C

***h = tan θ d h = 20sec2 θ = 20***

***20 d θ cos2 θ***

***d θ = d θ × d h = cos2 θ × – 2 when h = 5 , tan θ = 5 so θ = 140 or 0.244rad***

***d t d h d t 20 20***

 ***= 0.941 × – 2 = – 0.0941 rad/sec***

 ***20***

7. A car C is travelling from A to B along this road at a constant velocity

 of 30 m/s

 A B

 C

 θ

P

The car is being filmed by a camera at P which is 40 metres from A.

Find the rate of rotation of the camera in rads/sec when θ = π/3

***dx = 30 x = 40 tan θ***

***dt dx = 40 sec2 θ = 40***

 ***dθ cos2θ***

***dθ = dθ dx = cos2θ × 30***

***dt dx dt 40***

 ***= (½) 2× 30 = 0.1875 rad/sec***

 ***40***

8. A TV camera is set up 20 metres from the side of a motorcycle racetrack

 as shown below.

 The camera rotates so as to keep pointing at the rider as he races.

 🏍 *x* A

 20m

 θ

 Camera

The speed of the rider on this straight section of the track is 45 m/s.

Find the rate at which the camera is rotating at the instant the rider is

50 metres from A.

***dx = 45 m/s***

***dt***

***tan θ = 50 so cos θ ≈ .371***

 ***20***

***x = 20 tan θ***

***dx = 20 sec2 θ***

***dθ***

***dθ = d θ × dx = cos2 θ × 45 = .1379 × 45 = 0.31 rad/s***

***dt dx dt 20 20***

9.

80m

 θ

 s

The elevation of the sun is increasing at a rate : *dθ* = π rads per hour

 *dt*  9

Find the rate at which the length of the building’s shadow, S, is changing when θ = π . The height of the building is 80 metres.

 4

***dθ = π 80 = tan θ s = 80 (tan θ) – 1***

***dt 9 s s = 80 cot θ***

 ***ds = – 80 cosec2 θ***

 ***dθ***

***ds = ds × d θ***

***dt dθ dt***

 ***= – 80 cosec2 θ × π***

 ***9***

 ***= – 80 cosec2 (π/4) × π***

 ***9***

 ***= – 80 × 2× π***

 ***9***

 ***= – 160 π metres/hour = – 55.85 m/hr (shadow’s length is decreasing.)***

 ***9***

10. A large spherical balloon is being pumped up so that its surface area is

 increasing at a constant rate of 0.5 m2  per minute.

 Calculate the rate at which the volume is increasing when the radius is 1.5 m.

i.e. find *dV* when *r* = 1.5 m

 *dt*

N.B. Volume of a sphere = *4 π r3* and Surface area = *4 π r2*

 *3*

 ***dS = 0.5 dV = 4πr 2  dS = 8 πr***

 ***dt dr dr***

 ***dV = dV × dr × dS***

 ***dt dr dS dt***

 ***= 4πr2 × 1 × 0.5***

 ***8 πr***

 ***= 0.25 r = 0.375 m3/min***

11. The cone starts off full of water.

 Water is leaking out through a small hole in the bottom

Find the RATE at which the circular top of the water is changing when the depth of water has reached h = 5cm.

Hints: Use simple proportion to find a relationship between r and h.

The surface area is S = πr2

The volume of a cone is V = πr2h

 3

You are given dV = 2

 dt

You need to find dS

 dt

 at a constant rate of 2 cm3/min.

 10cm

 r

 40cm

 h

**r = 10**

**h 40**

**h = 4r**

**V = πr2h = πr24r = 4πr3**

 **3 3 3**

**dV = 4πr2**

**dr**

**dS = 2πr**

**dt**

**dS = dS × dr × dV**

**dt dr dV dt**

 **= 2πr × 1 × 2**

 **4πr2**

 **= 1**

 **r (if h = 5 then r = 5 )**

 **= 4 cm2/min 4**

 **5**

12. A conical rain gauge with radius 90mm and depth 180mm is filled with water at a constant rate of 150 000mm cubed/s. At what rate is the depth of the water increasing when the depth is 100mm?

***To find dh***

 ***dt***

 *90mm*

 *r = 90*

 *r h 180*

 *180 r = h*

 *h 2*

*dV = 150,000 mm3/sec*

*dt*

*V = π r2 h = π h2 h = π h3*

 *3 3 × 4 12*

*dV = π h2*

*dh 4*

*dh = dh × dV*

*dt dV dt*

 *= 4 × 150,000*

 *π h2*

 *= 4 × 150,000*

 *π 1002*

 *= 60 mm/sec*

 *π*