ASSIGNMENT 15 : INTEGRAL CALCULUS.

1.Write down the integrals of:

(a) $\int x^5\,dx$

(b) $\int 7x^6\,dx$

(c) $\int 4x\,dx$

(d) $\int x^{-3}\,dx$

(e) $\int x^{\frac{2}{3}}\,dx$

(f) $\int 2\,dx$

(g) $\int \frac{1}{x^2}\,dx$

(h) $\int \frac{3}{4x^6}dx$

(i) $\int \frac{1}{x}\,dx$

(j) $\int e^x\,dx$

(k) $\int \sin x\,dx$

(l) $\int \cos x\,dx$

(m) $\int \sec^2 x\,dx$

(n) $\int \operatorname{cosec}^2 x\,dx$

2. Find the integrals of :

(a) $\int e^{7x}\,dx$

(b) $\int \cos 6x\,dx$

(c) $\int \sin(3x-4)\,dx$

(d) $\int \sec^2(2x)\,dx$

(e) $\int \sec x \tan x\,dx$

(f) $\int \operatorname{cosec}3x \cot 3x\,dx$

(g) $\int x + \frac{1}{x}\,dx$

(h) $\int x^3 - \frac{1}{x^3}\,dx$

(i) $\int \sqrt{x}\,dx$

(j) $\int 4\pi^2 + 3x^4\,dx$

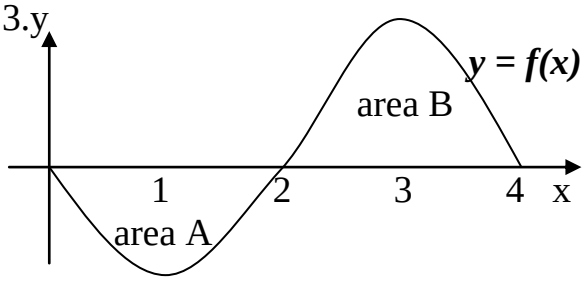
(k) $\int \frac{1}{\sqrt{x}}\,dx$

(l) $\int \frac{6x+5}{x}\,dx$

(m) $\int \frac{1}{(x-5)}\,dx$

(n) $\int \frac{x^5+\sqrt{x}}{x^2}\,dx$

3.



If area A = 3 cm^2 and area B = 5 cm^2 write down the values of:

(a) $\int_0^2 f(x)\,dx$

(b) $\int_2^4 f(x)\,dx$

(c) $\int_0^4 f(x)\,dx$

(d) What is the area between the graph $y = f(x)$ and the x axis between $x = 0$ and $x = 4$?

4. Find the area between $y = 3x(2-x)$ and the x axis from $x = 0$ to 2

5. Find the area between $y = e^x$ and the x axis from $x = 1$ to 3.

6. Find the area between $y = \cos x$ and the x axis from $x = 0$ to $\frac{\pi}{2}$.

7. Find the area between $y = \sec^2 x$ and the x axis from $x = 0$ to $\frac{\pi}{4}$.

8. Find the area between the curves $y = 2x^2$ and $y = x(3-x)$ (hint: find the intersection first)

9 Find the area between the curve $y = \frac{1}{x}$ and the **x** axis from $x = 2$ to 3

SOLUTIONS (15)

1.(a) $\int x^5\,dx$
$= \frac{x^6}{6} + c$

(b) $\int 7x^6\,dx$
$= x^7 + c$

(c) $\int 4x\,dx$
$= 2x^2 + c$

(d) $\int x^{-3}\,dx$
$= \frac{x^{-2}}{-2} + c$

(e) $\int x^{\frac{2}{3}}\,dx$
$= \frac{3}{5}x^{1\frac{2}{3}} + c$

(f) $\int 2\,dx = 2x + c$

(g) $\int x^{-2}\,dx$
$= -x^{-1} + c$

(h) $\int \frac{3x^{-6}}{4}dx$
$= \frac{\mathbf{3x^{-5}}}{-20} + c$

(i) $\int \frac{1}{x}\,dx$
$= \log_e x + c$

(j) $\int e^x\,dx$
$= e^x + c$

(k) $\int \sin x\,dx$
$= -\cos x + c$

(l) $\int \cos x\,dx$
$= \sin x + c$

(m) $\int \sec^2 x\,dx$
$= \tan x + c$

(n) $\int \operatorname{cosec}^2 x\,dx$
$= -\cot x + c$

2. (a) $\int e^{7x}\,dx = \frac{e^{7x}}{7} + c$

(b) $\int \cos 6x\,dx$
$= \frac{\sin 6x}{6} + c$

(c) $\int \sin(3x - 4)\,dx$
$= -\frac{\cos(3x-4)}{3} + c$

(d) $\int \sec^2(2x)\,dx$
$= \frac{\tan 2x}{2} + c$

(e) $\int \sec x \tan x\,dx$
$= \sec x + c$

(f) $\int \operatorname{cosec} 3x \cot 3x\,dx$
$= \frac{-\operatorname{cosec} 3x}{3} + c$

(g) $\int x + \frac{1}{x}\,dx$
$= \frac{x^2}{2} + \log x + c$

(h) $\int x^3 - x^{-3}\,dx$
$= \frac{x^4}{4} + \frac{x^{-2}}{2} + c$

(i) $\int x^{1/2}\,dx$
$= \frac{2x^{3/2}}{3} + c$

(j) $\int 4\pi^2 + 3x^4\,dx$
$= 4\pi^2 x + \frac{3x^5}{5} + c$

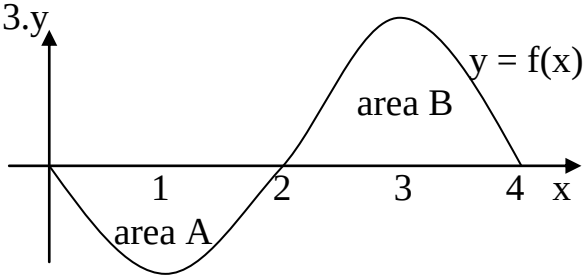
(k) $\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}} + c$

(l) $\int 6 + \frac{5}{x}\,dx$
$= 6x + 5\log x + c$

(m) $\int \frac{1}{(x-5)}\,dx$ **$= \log(x - 5) + c$**

(n) $\int \frac{x^5 + \sqrt{x}}{x^2}\,dx = \int x^3 + x^{-3/2}\,dx$
$= \frac{x^4}{4} - 2x^{-\frac{1}{2}} + c$

3.



If area A = 3 cm^2 and area B = 5 cm^2 write down the values of:

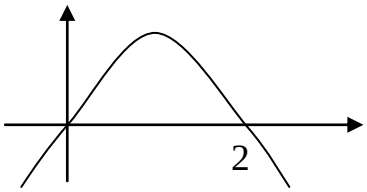
(a) $\int_0^2 f(x)\,dx = -3$

(b) $\int_2^4 f(x)\,dx = 5$

(c) $\int_0^4 f(x)\,dx = 2$

(d) What is the area between the graph y = f(x) and the x axis between x = 0 and x = 4 ? AREA = 8

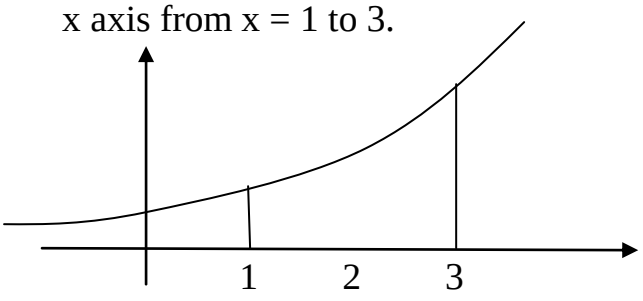
4. Find the area between $y = 3x(2 - x)$ and the x axis from x = 0 to 2



$A = \int_0^2 6x - 3x^2\,dx = \left[3x^2 - x^3\right]_0^2$

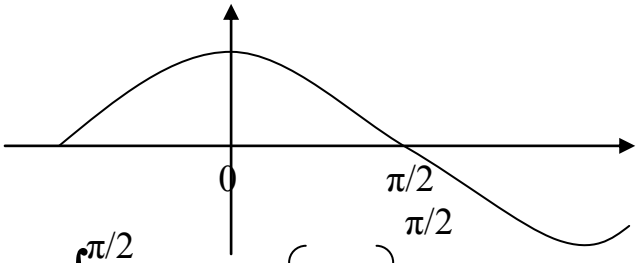
$= 12 - 8 = 4$ units^2

5. Find the area between $y = e^x$ and the x axis from x = 1 to 3.



$$A = \int_1^3 e^x \, dx = \left[e^x \right]_1^3 = e^3 - e$$

6. Find the area between y = cos x and the x axis from x = 0 to $\frac{\pi}{2}$.

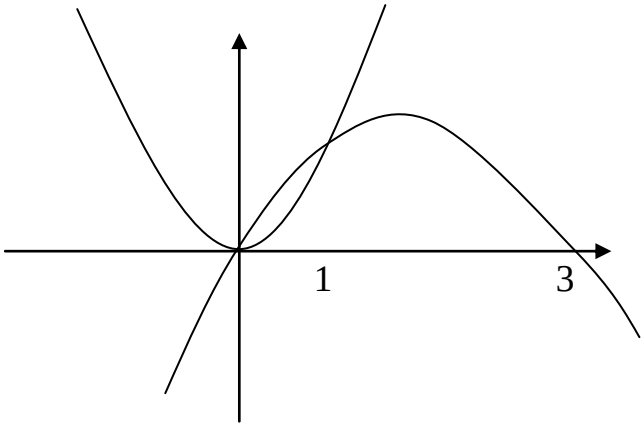


$$A = \int_0^{\pi/2} \cos x \, dx = \left[\sin x \right]_0^{\pi/2} = 1$$

7. Find the area between $y = \sec^2 x$ and the x axis from x = 0 to $\frac{\pi}{4}$.

$$A = \int_0^{\pi/4} \sec^2 x \, dx = \left[\tan x \right]_0^{\pi/4} = 1$$

8. Find the area between the curves $y = 2x^2$ and $y = x(3 - x)$ (hint: find the intersection first)



To find intersection :

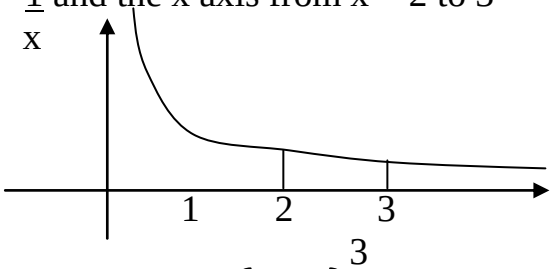
$$2x^2 = 3x - x^2$$
$$3x^2 - 3x = 0$$
$$3x(x - 1) = 0$$
$$x = 0, 1$$

$$\text{Area} = \int \mathbf{3}x - x^2 - 2x^2 \, dx$$

$$= \int_0^1 3x - 3x^2 \, dx$$

$$= \left[\frac{3x^2}{2} - x^3 \right]_0^1 = \frac{1}{2}$$

9 Find the area between the curve $y = \frac{1}{x}$ and the x axis from x = 2 to 3



$$\text{Area} = \int_2^3 1/x \, dx = \left[\log x \right]_2^3$$

$$= \log 3 - \log 2$$