QUADRATIC THEORY IN BRIEF.

Given $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

1. Use the quadratic formula to solve these equations (solutions to 2 dec pl.)

(a) $3x^2 + 9x + 5 = 0$

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{-9 \pm \sqrt{(9^2 - 4\times3\times5)}}{2\times3}$$

$$= \frac{-9 \pm \sqrt{(21)}}{6}$$ *(NB this is the "exact" answer!)*

x = –0.74 or –2.26 ***(NB this answer is only an approximation)***

(b) $5x^2 - 7x - 11 = 0$

$ax^2 + bx + c = 0$ *a = 5 but b = -7 and c = - 11*

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{+7 \pm \sqrt{(49 - 4\times5\times(-11))}}{2\times5}$$

$$= \frac{+7 \pm \sqrt{(49 - 4\times5\times(-11))}}{2\times5}$$

$$= \frac{+7 \pm \sqrt{(269)}}{10}$$ *(NB this is the "exact" answer!)*

x = 2.34 or –0.94 ***(NB this answer is only an approximation)***

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

THE DISCRIMINANT $\Delta = b^2 - 4ac$

REMEMBER: The solutions of an equation are where the graph of the equation crosses the x axis.

"COMPLETING THE SQUARE" METHOD.

2. Show clearly how to solve each of the following 4 equations by completing the square (even though 2 of them factorise)
and state how the **discriminant** affects the type of solutions.

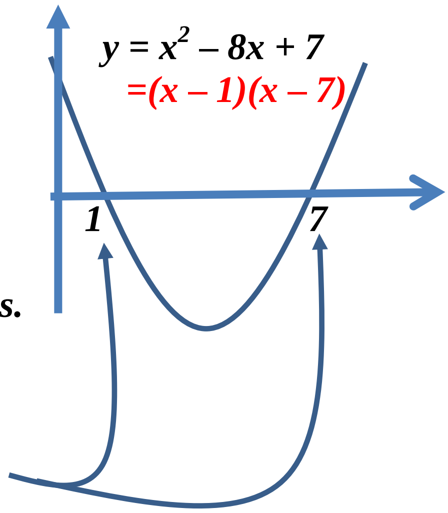
(a) $x^2 - 8x + 7 = 0$

$$\begin{aligned} x^2 - 8x &= -7 \\ x^2 - 8x + 16 &= -7 + 16 \\ (x-4)^2 &= 9 \end{aligned}$$

so $x - 4 = 3$ or $x - 4 = -3$

$x = 7$ or 1

The solutions are where the graph crosses the x axis.

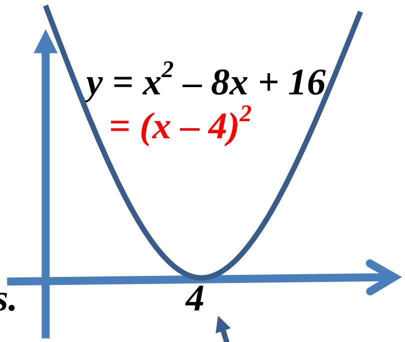


$$\begin{aligned} \Delta &= 8^2 - 4\times1\times7 \\ &= 64 - 28 \\ &= 36 \text{ (a perfect square) so we get 2 rational sols} \end{aligned}$$

(b) $x^2 - 8x + 16 = 0$

$$\begin{aligned} x^2 - 8x &= -16 \\ x^2 - 8x + 16 &= -16 + 16 \\ (x-4)^2 &= 0 \\ x &= 4 \end{aligned}$$

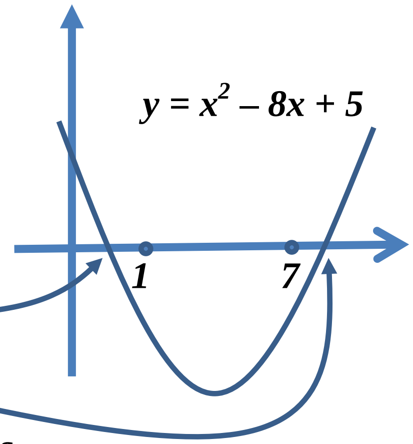
The solutions are where the graph crosses the x axis.



$$\begin{aligned} \Delta &= 8^2 - 4\times1\times16 \\ &= 64 - 64 \\ &= 0 \text{ so we get 1 rational sol.} \end{aligned}$$

(c) $x^2 - 8x + 5 = 0$

$$\begin{aligned}
x^2 - 8x &= -5 \\
x^2 - 8x + 16 &= -5 + 16 \\
(x-4)^2 &= 11 \\
x - 4 &= \pm\sqrt{11} \\
x &= 4 \pm \sqrt{11} \\
&\approx 7.32,\ 0.683
\end{aligned}$$



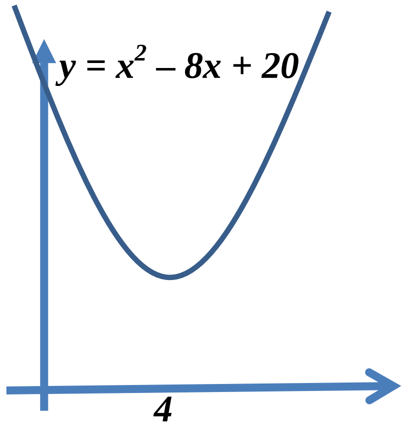
The solutions are where the graph crosses the x axis.

$\Delta = 8^2 - 4\times1\times5$
$= 64 - 20$
$= 44$ *so we get 2 irrational sols. because 44 does not have an exact square root.*

(d) $x^2 - 8x + 20 = 0$

$$\begin{aligned}
x^2 - 8x &= -20 \\
x^2 - 8x + 16 &= -20 + 16 = -4 \\
(x-4)^2 &= -4
\end{aligned}$$

Can't find $\sqrt{-4}$



The solutions are where the graph crosses the x axis but it does not cross the x axis so there are no real solutions.

$\Delta = 64 - 80 = -16$ *so no real sols.*

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3. The Discriminant is $\Delta = b^2 - 4ac$.
State what **type** of solutions you get if the discriminant is :

(a) 0
= 1 rat sol
(graph sits on x axis)

(b) 1 or 4 or 9 or 16 etc
= 2 rat sol
(graph crosses x axis at whole numbers or fractions)

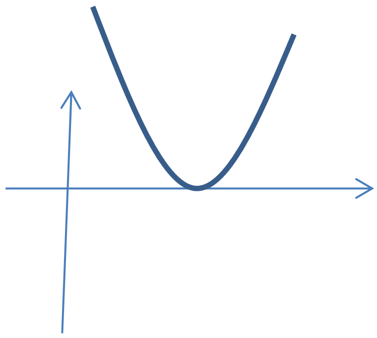
(c) 2 or 3 or 5 or 6 etc
= 2 irrat sol
(graph crosses x axis at numbers which are SURDS (eg √3))

(d) -1 or -5 or -76 etc
= NO real solutions
(graph does not cross x axis)

EXAMPLES

1.

Find the **value** of ***p*** so that
$x^2 - 10x + p = 0$ has one solution.



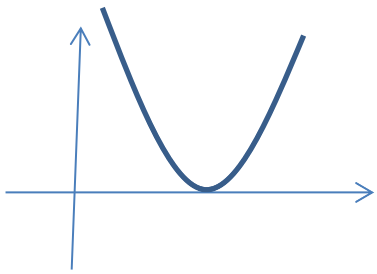
This will have only 1 solution if the graph sits on the ***x*** axis.
In which case, the discriminant = 0

$$\Delta = 100 - 4p = 0$$
$$100 = 4p$$
$$25 = p$$

Note: if p = 25, the equation is $x^2 - 10x + 25 = 0$
so that $(x-5)^2 = 0$
and the only solution is $x = 5$

2.

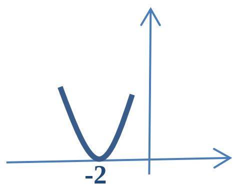
Find ***p*** so that $x^2 + (p+2)x + (3p-2) = 0$
has only one rational solution.



This will have only 1 solution if the graph sits on the ***x*** axis.
In which case, the discriminant = 0

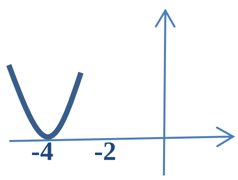
$$\Delta = (p+2)^2 - 4(3p-2) = 0$$
$$p^2 + 4p + 4 - 12p + 8 = 0$$
$$p^2 - 8p + 12 = 0$$
$$(p-2)(p-6) = 0$$
$$p = 2 \text{ or } 6$$

Some students find this "double" answer confusing:



It means that if p = 2 the equation $x^2 + (p+2)x + (3p-2) = 0$
becomes $x^2 + 4x + 4 = 0$
and THIS equation only has 1 solution (x = -2)

AND



It means that if p = 6 the equation $x^2 + (p+2)x + (3p-2) = 0$
becomes $x^2 + 8x + 16 = 0$
and THIS equation only has 1 solution (x = -4)