**ROOTS OF QUADRATICS AND COEFFICIENTS OF THE EQUATION.**

1. If the roots of an equation are 3 and 4 the equation is……

***(x – 3)(x – 4) = 0***

***x2 – 7x + 12 = 0***

notice (3 + 4) ( 3 × 4)

***(also notice the negative sign!)***

2. If the roots of an equation are 2 and -5 the equation is……

***(x – 2)(x + 5) = 0***

***x2 + 3x – 10 = 0***

notice (2 + -5) ( 2 × -5)

3. If the roots of an equation are α and β the equation is……

***(x – α)(x – β) = 0***

***x2 – (α + β )x + α β = 0***

notice (sum) (product)

4. We could just do this… If roots are 5 and 9, the sum = 14 and product is 45

so the equation ***is x2 – (sum)x + (product) = 0***

***x2 – 14 x + 45 = 0***

5. If roots are ¾ and ½ , the SUM is 5 and the product is 3

4 8

so the equation ***is x2 – (sum)x + (product) = 0***

***x2 – 5x + 3 = 0***

***4 8***

OR we could write this as ***8x2 – 10x + 3 = 0***

(incidentally, this factorises to ***(4x – 3)(2x – 1) = 0***

producing roots of ¾ and ½ of course)

6. Consider the equation ***x2 – 6x + 7 = 0***

***x2 – 6x = -7***

completing the square: ***x2 – 6x + 9 = -7 + 9***

***(x – 3)2 = 2***

***x – 3 = ±√2***

***x = 3 ±√2***

***The roots are 3 +√2 and 3 –√2***

***Sum of roots = 6***

***Product of roots = (3 +√2)( 3 –√2) = 7***

***Using the Theory as in Qu 3 the equation is…***

***(x – α)(x – β) = 0***

***x2 – (α + β )x + α β = 0***

(sum) (product)

***ie x2 – 6x + 7 = 0 (The theory works for irrational solutions too)***

***NB It is NOT necessary to write and expand the following if we know of this***

***technique…………..***

***x – (3 +√2) x – (3 –√2) = 0***

7. Now consider this SPECIAL CASE:

***x2 – 4x + 5 = 0***

***x2 – 4x = -5***

completing the square: ***x2 – 4x + 4 = -5 + 4***

***(x – 2)2 = -1***

***x – 2 = ±√-1***

***x = 2 ± √-1***

***Up until now, we have stopped here and said this equation has no solutions because we cannot evaluate √-1 in our number system but…….***

***if the ROOTS are 2 + √-1 and 2 – √-1***

***the SUM IS 4 AND THE PRODUCT IS (2 + √-1)( 2 – √-1) = 5 !!!***

***If the roots do not exist, how come they ADD to 4 and multiply to give 5?***

***(we usually write √-1 = i for convenience)***

***In fact they DO exist.***

***They are a new type of number called COMPLEX NUMBERS.***

Complex numbers are of the form ***a + bi*** where ***a*** and ***b*** are real and ***i = √-1***

**REAL NUMBERS** are those we can represent ON a NUMBER LINE

-3 -2 -1 0 1 2 3

**COMPLEX NUMBERS** canbe represented on a **NUMBER PLANE**

**Imaginary axis**

The number shown here is

***2 + 3i***

And may be represented by

A DOT at (2, 3) or by the vector from (0, 0) to (2, 3).

This is called an ARGAND diagram (or plane) named after the person who 1st thought of it.

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|  |  |  |  | **Real axis** |
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**ADDITION.** On a number line, we add numbers like vectors **on** the line

Example: 3 + -4 = -1

-3 -2 -1 0 1 2 3

We add complex numbers just like vectors in a plane.

Example:

***If u = 3 + i and v = 1 + 2i***

***then u + v = 4 + 3i***

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To **SUBTRACT** complex numbers we simply ADD the opposites as with Integers.

Example***: (4 + 3i) – (2 + 5i)***

We put ***(4 + 3i) + (-2 + -5i)***