**ASSESSMENT SCHEDULE**

#### **91577 Apply the algebra of COMPLEX NUMBERS in solving problems**

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| --- | --- | --- |
| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply the algebra of complex numbers in solving problems* involves:  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply the algebra of complex numbers, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing **limited knowledge of algebra of complex numbers in solving problems.**

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 1a | ***z2 – 8z + 16 = 16 – 22***  ***(z – 4)2 = - 6***  ***z – 4 = ±i√6***  ***z = 4 ±i√6*** | Correct expression. |  |  |
| 1b | ***(5 – 2i)(3 – 4i) = 7 – 26i*** | Correct solution. |  |  |
| 1c | ***x – k = 25 + x – 10 √x***  ***10√x = 25 + k***  ***√x = 25 + k***  ***10***  ***x = ( 25 + k )2***  ***100*** | Award u if minor error | Award r only for  Correct solution. |  |
| 1d | ***Let z rcisθ z3 = - n12***    ***r3cis 3θ= n12 cis(180 + 360n)***    ***r3 = n12 3θ=180 + 360n***  ***r = n4 θ = 60 + 120n***  ***z1 = n4 cis 600***  ***z2 = n4  cis 1800***  ***z3 = n4 cis 3000*** | Award u if changed correctly into polar form:  r3cis 3θ=  n12 cis(180+360n)  either using degrees or radians. | Correct solution either using degrees or radians. |  |
| 1e | ***x2 + 8x + 16 = 4(x + p)***  ***x2 + 4x + (16 – 4p) = 0***  ***2 real sols is > 0***  ***16 – 4(16 – 4p) > 0***  ***16 – 64 + 16p > 0***  ***16p > 48***  ***p > 3*** | Award u if  ***x2 + 8x + 16 = 4(x + p)***  is obtained | Award r if student gets quadratic in the form:  ***x2 + 4x + (16 – 4p) = 0*** | Correct solution showing correct logical steps with correct mathematical statements. |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 2a | (4 + √3) × (5 + √3) = 23 + 9√3  (5 – √3 ) (5 + √3) 22 22 | Correct expression.  (*reluctantly allow*  *23 + 9√3*  *22* |  |  |
| 2b | ***w = 6 – 2i correctly placed at (6, -2)***  ***z = 4 – 2i correctly placed at (4, -2)*** | If ***w*** and ***z*** are correctly worked out but not placed on Argand diagram award u | BOTH correctly placed as points on Argand plane = r |  |
| 2c | ***3 + 2i is a root and so is 3 – 2i***  ***Let 3rd root (real) = b***  ***(z – b)(z2 – 6z + 13) = 0***  ***Comparing number terms: -13b = -26***  ***So b = 2***  ***Comparing terms in z2: -8z2 = -kz2***  ***So k = 8***  ***Solutions are 3 ± 2i and 2*** | Award u if the  root 3 – 2i is stated | Correct solution. |  |
| 2d | ***w = ( x + yi) × (x – 6) – i (y – 8)***  ***(x – 6) + i(y – 8) (x – 6) – i (y – 8)***  ***= x(x – 6) + y(y – 8) + i(y(x – 6) – x(y – 8))***  ***(x – 6)2 + (y – 8)2***  ***If purely imag then Re = 0 so…***  ***x(x – 6) + y(y – 8) = 0***  ***x2 – 6x + y2 – 8y = 0***  ***x2 – 6x + 9 + y2 – 8y+ 16 = 25***  ***or***  ***(x – 3)2 + (y – 4)2 = 25***  ***(Locus is a circle, centre (3, 4) radius 5)*** | Award u for arranging as  ***( x + yi)***  ***(x – 6) + i(y – 8)*** | Award r for mult by  ***(x – 6) – i (y – 8)***  ***(x – 6) – i (y – 8)***  ***correctly*** | Correct solution.  Exceptions  z ≠ 6 + 8i or 0 + 0i **not required** for excellence.  ***Stating that:***  ***“Locus is a circle, centre (3, 4)***  ***radius 5”***  ***is optional.***  **Must get correct equation though.** |
|  |  |  |  |  |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 3a | ***R = f(2) = 20*** | Correct answer required for u |  |  |
| 3b | ***6cis (6θ) = 3cis(4θ)***  ***2cis(2θ)*** | Correct solution.  required for u |  |  |
| 3c | ***Let roots = α andβ***  ***So α +β = -6 and αβ = 12***  ***If roots are 4α and 4β***  ***Sum = -24 and product = 16×12 =192***  ***Equ is x2 + 24x + 192 = 0*** | Correct sum and product of roots of original equ  required for u | Correct solution  required for r |  |
| 3d | ***z = rcisθ = 2cis(300 )***  ***z5 = 25cis(1500)***  ***= 25( -√3 + i ) = -16√3 + 16i***  **2 2** | Correct polar form and use of De Moire’s theorem for u | Correct **exact** solution for r. |  |
| 3e | ***ln (3x – 2) = ln k2***  ***(x – 5)***  ***(3x – 2) = k2***  ***(x – 5)***  ***3x – 2 = k2(x – 5)***  ***3x – 2 = k2x – 5k2***  ***5k2 – 2 = k2x – 3x***  ***5k2 – 2 = x(k2 – 3)***  ***x = 5k2 – 2***  ***k2 – 3*** | Correct 2nd line for u | Correct equ for r  No restrictions needed. |  |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.* |
| 3f | ***Let z = x + iy***  ***x + iy***  ***(x – 4) + iy***  ***x + iy × (x – 4) – iy line 2***  ***(x – 4) + iy (x – 4) – iy***  ***= x(x – 4) + y2 + i( y(x – 4) – yx ) line 3***  ***(x – 4)2 + y2 (x – 4)2 + y2***  ***If Re = Im***  ***x(x – 4) + y2 = y(x – 4) – yx***    ***-4y = x2 – 4x + y2***  ***0 = x2 – 4x + y2 + 4y***  ***8 = x2 – 4x +4 + y2 + 4y + 4***  ***Locus is: (x – 2)2 + (y + 2)2 = 8***  ***Locus is a circle, centre (2, -2) radius √8*** | Award u up to 2nd  Line. | Award r for getting 3rd line | Award t for correct locus.  Restrictions (0, 0) and (4, 0) not required for t. |

**ASSESSMENT SCHEDULE**

#### **91578 Apply DIFFERENTIATION methods in solving problems**

**IN RECENT YEARS, STUDENTS HAVE NOT BEEN ASKED TO DO A 1st or 2nd DERIVATIVE TEST.**

**OBVIOUSLY WE SHOULD BE TEACHING THIS AND I HAVE CHANGED QU 1(c) SO AS TO INCLUDE IT IN THE EXAM THIS YEAR.**

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| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply differentiation methods in solving problems* involves*:*  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply differentiation methods, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply differentiation methods, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question demonstrating limited knowledge of differentiation techniques

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) |  |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 1a | ***yꞌ = tan(x2)×2sinxcosx – sin2x×sec2(x2)×2x***  ***( tan(x2) )2*** | Correct derivative. |  |  |
| 1b | ***yꞌ = (x – 1)(2x – 1) – (x2 – x + 9) = 0 at st.pt***  ***(x – 1)2***  ***2x2 – 3x + 1 – x2 + x – 9 = 0***  ***x2 – 2x – 8 = 0***  ***(x + 2)(x – 4) = 0***  ***x = -2 , 4*** | Correct derivative = u | Correct derivative with correct solution = r. |  |
| 1c | ***yꞌ = x × 1 + ln(x) – 3 = 0 at max/min point***  ***x***  ***so ln(x) – 2 = 0***  ***ln(x) = 2***  ***x = e2 ≈ 7.4***  **1st *deriv. test or 2nd deriv. test***  ***yꞌꞌ = 1/x***  ***if x = 7.4***  ***yꞌꞌ > 0 so min***   |  |  |  |  | | --- | --- | --- | --- | | ***x*** | ***7*** | ***7.4*** | ***8*** | | ***yꞌ*** | ***-*** | ***0*** | ***+*** |   ***min*** | Correct derivative = u | Correct derivative with ***x = e2*** and either 1st or 2nd deriv. test correct solution = r |  |
| 1d | ***dy = 3y2 – 12 dx = 3t2 – 27***  ***dt dt***  ***dy = 3t2 – 12***  ***dx 3t2 – 27***  ***if tan is vert denominator = 0***  ***3t2 – 27 = 0 so 3(t2 – 9 ) = 0 t = ±3***  ***If t = 3 y = -9, x = - 54 ( -54, -9)***  ***If t = -3 y = 9, x = 54 (54, 9)*** | Correct dy for u  dx  (or just the fact that dx = 0 )  dt | Correct answer for for r |  |
| 1e | h = 12  πr2  S = 2πr2 + 2πr × 12  πr2  = 2πr2 + 24  r  Sꞌ = 4πr – 24r – 2  = 4πr – 24 = 0  r2  4πr = 24  r2  r3 = 6 r ≈ 1.24m h ≈ 2.48m  π    Min S ≈ 29 m2 | Correct S value for u | Correct derivative and r value for r | Correct r and h and min S found for t. |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 2a | ***yꞌ = - sin(3x2 – 4) × 6x*** | Correct derivative. |  |  |
| 2b | ***yꞌ = 2e2x + 4***  ***e2x + 4x***  ***sub x = 0***  ***yꞌ = 6*** | Correct derivative and sub x = 0 to get 6  for u |  |  |
| 2c | ***yꞌ = (x2+ x – 2) e same = 0***  ***so x2 + x – 2 = 0 (as ef(x) is always > 0)***  ***(x + 2)(x – 1) = 0***  ***x = - 2, 1*** | Correct .  for u | Correct derivative with correct solution for r. |  |
| 2d | ***dx = 3cost dy = - 4sint***  ***dt dt***  ***dy = - 4sint***  ***dx 3cost***  ***If t = 450 dy = - 4***  ***dx 3***  ***so grad of normal = + 3***  ***4*** | Correct grad of tan | Correct grad of normal |  |
| 2e | ***dV = 6 V = 4πr3 A = 4πr3***  ***dt 3***  ***dV = 4πr3 dA = 8πr***  ***dr dr***  ***dA = dA × dr × dV***  ***dt dr dV dt***  ***= 8πr × 1 × 6 = 12 = 12 cm2/s***  ***4πr3 r 20***  ***= 0.6 cm2/s*** | Correct derivatives  dV dA dV  dr dr dt  for u | Correct chain of derivatives for r | Correct derivatives with correct solution for t |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.* |
| 3a | ***y = (10 + x5)  1/5***  ***yꞌ = 1 (10 + x5) -4/5 × 5x4***  ***5*** | Correct derivative. |  |  |
| 3b | ***yꞌ = 4(3x – x2)3 ( 3 – 2x)***  ***sub x = 1 yꞌ = 32 y = 16***  ***equ of tan is of the form y = mx + c***  ***16 = 32 + c***  ***-16 = c***  ***equ of tangent is y = 32x – 16*** | Correct derivative with correct solution. |  |  |

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| 3c | ***y = x4 – 12x2***  ***yꞌ= 4x3 – 24x = 0 at max/min***  ***4x(x2 – 6)= 0***  ***x = 0 and ±√6***  ***yꞌꞌ= 12x2 – 24 = 0 at infl pts***  ***12(x2 – 2) = 0***  ***x = ±√2***  ***concave down if -√2 < x < √2*** | Correct ***yꞌ and***  ***yꞌꞌ*** for u | Some justification  For why it is concave down not concave up.  ie Sketch graph  or a 1st or 2nd deriv. test.  Correct solution  for r. |  |
| 3d | ***(i) 1 x = 7***  ***2 x = 3, 7 , 11, 15***  ***3 5 < x < 7 and 11 < x < 15***  ***4. x = 5 and x < 3***  ***(ii) 6***  ***(iii) 6***  ***(iv) 3***  ***(v) no lim*** | 4 correct for u | 6 correct for r |  |
| 3e | A D  ***x x√2***  ***B d C***  ***d + x + x√2 = 40***  ***d = 40 – x – x√2***  ***Area A = xd + x2***  ***2***  ***A = 40x – x2 – x2√2 + x2***  ***2***  ***dA = 40 – 2x – 2x√2 + x = 0 for max***  ***dx***  ***40 – x – 2x√2 = 0***  ***40 = x ( 1 + 2√2)***  ***x = 40 ≈ 10.45 cm***  ***(1 + 2√2)***  ***AB = 10.45 cm***  ***CD = 14.78 cm***  ***BC = 14.78 cm*** | Correct equ for cross section = u | Correct derivative of equ for A and put equal to zero  = r | Correct derivatives with correct solution.  Finding max cross-sectional area not required but  ***max cross section is:≈ 209 cm2*** |

**ASSESSMENT SCHEDULE**

#### **91579 Apply integration methods in solving problems**

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| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| *Apply integration methods in solving problems* involves*:*  • selecting and using methods  • demonstrating knowledge of concepts  and terms  • communicating using appropriate  representations. | *Apply integration methods, using relational thinking*, *in solving problems* involves one or more of:  • selecting and carrying out a logical  sequence of steps  • connecting different concepts or  representations  • demonstrating understanding of  concepts  • forming and using a model;  and also relating findings to a context, or  communicating thinking using appropriate mathematical statements. | *Apply integration methods, using extended abstract thinking, in solving problems* involves one or more of:  • devising a strategy to investigate or solve  a problem  • identifying relevant concepts in context  • developing a chain of logical reasoning, or  proof  • forming a generalisation;  and also using correct mathematical statements, or communicating mathematical insight. |

**PLEASE BE AWARE THAT THIS PAPER DOES NOT INCLUDE SEPARATION OF VARIABLES METHOD FOR DIFFERENTIAL EQUATIONS AND SO SOME DOUBLING UP OF AREA PROBLEMS HAS BEEN NECESSARY.**

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing limited knowledge of integration techniques.

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | **Not Achieved** | **Achievement** | | | **Achievement with Merit** | | | **Achievement with Excellence** | | |
| **Score range** | | | 0 - 6 | 7 - 13 | | | 14 - 18 | | | 19 -24 | | |
| Question ONE | | | Evidence | | | Achievement (u) | | | Merit (r) | | | Excellence (t) | | |
|  | | |  | | | *Apply integration methods in solving problems.* | | | *Apply integration methods, using relational thinking, in solving problems.* | | | *Apply integration methods, using extended abstract thinking, in solving problems.* | | |
| 1a | | | ***-cos(4x) + c***  ***4*** | | | Correct integration. | | |  | | |  | | |
| 1b | | | ***2***  ***= e2x = e4 – e2 ≈ 23.6***  ***2 1  2 2*** | | | Correct integration. | | |  | | |  | | |
| 1c | | | ***u = x3 + 4x + 3 so du = (3x2 + 4)dx***  = ***ln (u) + c***  ***k***  ***= ln(x3 + 4x + 3)***  ***0***  ***= ln(k3 + 4k + 3) – ln(3)*** | | | Obtains log function = u | | | Correct integration with correct solution. | | |  | | |
| 1d | | | ***k = k + 1 – x***  ***x***  ***k = (k + 1)x – x2***  ***x2 – (k + 1)x + k = 0***  ***(x – 1)(x – k) = 0***  ***x = 1, k***  A =  ***k***  ***= kx + x – x2 – kln(x)***  ***2 1***  ***= k2 + k – k2 – kln(k) – k + 1 – ½ - ln(1)***  ***2***  ***= k2 – ½ – kln(k)***  ***2*** | | | Correct intersection  = u | | | Correct integration evaluated = r | | |  | | |
| 1d | | | A  Intersection points: ***x3 – 4x = x2 – 2x***  B  ***x3 – x2 – 2x = 0***  ***x(x2 – x – 2) = 0***  ***x(x + 1)(x – 2) = 0***  ***intersection points x = -1, 0, 2***  Area A = cubic – parabola  = ) dx  ) dx  ***0***  ***= x4 – x2– x3***  ***4 3 -1***  ***=***  Area B = parabola –cubic    ) ***dx***  ***2***  ***= x4 – x2– x3***  ***4 3 0***  ***= – x4 – x2 + x3  = 8***  ***4 3 3***    ***Total area between curves = 3.083 or 37***  ***12*** | | | Correct intersection for u | | | Correct idea of area = area under cubic – area under parabola (and vice versa) shown with equations. | | | Correct solution showing correct integrations and correct mathematical logic and statements. | | |
| Question TWO | | Evidence | | | | | Achievement (u) | | | Merit (r) | | | Excellence (t) | |
|  | | |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *x* | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | | *f(x)* | 1 | 3 | 7 | 13 | 15 | 11 | 5 | 4 | 2 | |  | y0 | y1 | y2 | y3 | y4 | y5 | y6 | y7 | y8 |   *A =2 1+ 2 + 4(3 + 13 +11+4) + 2(7+15+5*  *3*  *= 120.7* | | | | | *Apply integration methods in solving problems.* | | | *Apply integration methods, using relational thinking, in solving problems.* | | | *Apply integration methods, using extended abstract thinking, in solving problems.* | |
| 2a | | Correct calculation. | | |  | | |  | |
| 2b | | A =  = ***ln(2)*** | | | | | Correct integration = u.  accept  ln(2k) – ln(k) | | | Award r for ***ln(2)*** | | |  | |
| 2c | | ***Intersection points when x2 = x + 6***  ***x2 – x – 6 = 0***  ***(x – 3) (x + 2) = 0***  ***x = 3, -2***  ***A =***  ***3***  ***= x2 + 6x – x3 = 20.83***  ***2 3 -2*** | | | | | Correct intersection and correct “idea” integral of  line - parabola | | | Correct integration with correct solution. | | |  | |
| 2d | | ***yꞌ = 12(x – 2) – 3***  ***y = 12 (x – 2) – 2  + c***  ***-2***  ***y = -6 + c***  ***(x – 2)2***  ***Sub x = 3, y = 4 4 = -6 + c so c = 10***  ***y = - 6 + 10***  ***(x – 2)2***  ***Sub x = 4 y = 8.5*** | | | | | Correct integration with correct constant. | | | Correct integration with correct constant and ***y*** value found. | | |  | |
| 2e | | ***Let u = 4x + 1 so du = 4dx***  ***Also u – 1 = 4x so 2x = u – 1***  ***2***  I = =  = =  =  = + c  = + c | | | | | Correct substitution ready to integrate = u | | | Correct manipulations needed to integrate = r | | | Correct solution  = t  Further simplification not necessary. | |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.* |
| 3a(i) | .  ***= 2x2 + 3x + 2ln*|*x*| *+ c*** | Correct integration.  **|*x*|** not essential |  |  |
| 3a(ii) | ***= ex*** – ***e – 2x*** + c  2 | Correct integration. |  |  |
| 3b | ***yꞌ = 6t2 + 6t + c***  ***sub yꞌ = 2, t = 1 so c = -10***  ***yꞌ = 6t2 + 6t – 10***  ***so y = 2t3 + 3t2 – 10t + d***  ***sub t = 1, y = 3 so d = 8***  ***y = 2t3 + 3t2 – 10t + 8***  ***sub t = 3, y = 59*** | 2 Correct integrations. | Correct equation for ***y*** and correctly evaluated ***y*** when t = 3 |  |
| 3c | ***y =* 1 *+ 4 best idea***  ***(x + 2)***  ***=***  ***2***  ***= x + 4ln(x + 2)***  ***0***  ***= 2 + 4ln(4) – 4ln(2)***  ***= 2 + 4ln(2)***  ***\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_***  ***OR let u = x + 2 so x + 6 = u + 4***  ***du = dx***  ***I =***  ***=***  ***4***  ***= u + ln(u) = 4 + 4ln4 – 2 – 4ln2***  2  ***= 2 + 4ln(2)*** | Correct integration = u | Correct  Integration evaluated = r. |  |
| 3d | Need to find equations of curves first.  ***Hyp is y = 8***  ***x***  ***parab is y = -ax2+ bx***  ***sub(2, 4) 4 = -4a + 2b***  ***sub(4, 2) 2 = -16a + 4b***  ***solving a = ¾ b = 3 ½***  ***parab is y = - ¾ x2 + 3 ½ x***  ***area is***  ***= x -***  ***4***  ***= - x3 + 7x2 – 8ln*|*x*|**  4 4 2  ≈ 1.45 |  | Finding both equations which is needed in order to do any integration = r | Correct integration  = t  **|*x*|** not required  Correct integration with correct logical steps and correct solution. |