***Solutions (Roots) of Quadratics.***

*If the roots of a quadratic are α and β then the equation must be :*

*(x – α) (x – β) = 0*

*Multiplying out, we get:*

*x2 – αx – βx + αβ = 0*

*x2 – (α + β) x + αβ = 0*

*so for* ***any quadratic equation in the form x2 + bx + c = 0***

***the PRODUCT of the roots αβ = c***

***and the SUM of the roots α + β = – b***

*eg1. for the equation x2 + 9x + 20 = 0*

*αβ = 20*

*α + β = –9*

*eg2. for this equation x2 – 5x – 6 = 0*

*αβ = –6*

*α + β = +5*

*eg3. for the equation:3x2 – 5x + 11 = 0*

*first we should divide by 3*

*x2 – 5 x + 11 = 0*

*3 3*

*so* *αβ = 11*

*3*

*α + β = 5*

*3*

*1. Suppose we say that the roots of the equation x2 – 3x + 2 = 0 are α and β*

*then FIND the equation with roots of*

*2α and 2β.*

***Firstly*** ***αβ = 2***

***and α + β = 3***

*The equation would be :*

*(x – 2α) (x – 2β) = 0*

*Multiplying out, we get:*

*x2 – 2αx – 2βx + 4αβ = 0*

*x2 – 2(α + β) x + 4αβ = 0*

*subs αβ = 2*

Or we just say:

Product = 2α×2β

=4×αβ = **8**

Sum =2α+2β

=2(α+β)= **6**

*and α + β = 3*

*we get :*

*x2 – 2(3)x + 4(2) = 0*

*x2 – 6x + 8 = 0*

*2. Suppose we say that the roots of the equation x2 – 3x + 2 = 0 are α and β*

*then FIND the equation with roots of*

***3α*** *and* ***3β.***

*Firstly* *αβ = 2*

*and α + β = 3*

*The equation would be :*

*(x – 3α) (x – 3β) = 0*

*Multiplying out, we get:*

*x2 – 3αx – 3βx + 9αβ = 0*

*x2 – 3(α + β) x + 9αβ = 0*

Or we just say:

Product = 3α×3β

=9×αβ = **18**

Sum =3α+3β

=3(α+β)= **9**

*subs αβ = 2*

*and α + β = 3*

*we get :*

*x2 – 3(3)x + 9(2) = 0*

*x2 – 9x + 18 = 0*

*3. Suppose we say that the roots of the equation* ***x2 + bx + c =*** *0 are α and β*

*then FIND the equation with roots of*

***4α*** *and* ***4β.***

*Firstly* *αβ = c*

*and α + β = –b*

*The equation would be :*

Or we just say:

Product = 4α×4β

=16×αβ

= **16c**

Sum =4α+4β

=4(α+β)

= **-4b**

*(x – 4α) (x – 4β) = 0*

*Multiplying out, we get:*

*x2 – 4αx – 4βx + 16αβ = 0*

*x2 – 4(α + β) x + 16αβ = 0*

*subs αβ = c*

*and α + β = –b*

*we get :*

*x2 – 4(-b)x + 16(c) = 0*

***x2 + 4bx + 16c = 0***