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Some athletes abuse human growth hormone and/or steroids to build muscle and strength. More and more sophisticated tests are needed to detect people who have these in their system. The best tests involve urine samples.

It is a well-established fact that the body will eliminate such substances at a rate proportional to the amount present at any particular time.

After taking such a substance several weeks ago, an athlete decides to have his urine tested and the amount detected was 120 $\mu\text{g}/100\text{mL}$ of urine.

He did not take any more of the substance.

Two months later he was tested again and the reading was 95 $\mu\text{g}/100\text{mL}$.

- (a) Find a formula for the amount of the substance in his urine at any time t months.
(b) Find what the reading would be expected to be after $t = 6$ months.
(c) If the substance can still be detected when the reading is as low as 20 $\mu\text{g}/100\text{mL}$ find how long it will be before he can enter competitions without being detected.

$$\frac{dS}{dt} = kS \quad \text{so that} \quad \int \frac{dS}{S} = \int k dt$$

$$\ln(S) = kt + c$$

$$\text{subs } t = 0, S = 120 \text{ so } \ln(120) = 0 + c$$

$$\ln\left(\frac{S}{120}\right) = kt$$

$$\text{Subs } t = 2, S = 95$$

$$\ln\left(\frac{95}{120}\right) = 2k \quad \text{so } k = -0.1168$$

Equ becomes:

$$\ln\left(\frac{S}{120}\right) = -0.1168t \quad \text{EQU 1}$$

$$S = 120 e^{-0.1168t} \quad \text{EQU 2}$$

$$\text{(b) subs } t = 6 \text{ in EQU 2 so } S = 120 e^{-0.1168 \times 6} \approx 59.5 \mu\text{g}$$

$$\text{(c) subs } S = 20 \mu\text{g in EQU 1 so } \ln\left(\frac{20}{120}\right) = -0.1168t$$
$$t \approx 15.3 \text{ months!}$$

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