**MATHEMATICS WITH CALCULUS**

**2013**

**TIME ALLOWED: 3 Hours**

# Show ALL working for ALL questions

**If you need more space, use the back pages of each assessment.**

**Unless otherwise stated, 3 significant figure numerical answers will be adequate.**

**Mathematical formulae and tables are supplied.**

|  |  |
| --- | --- |
| **Achievement Standard Assessment** | **Grade** |
| Complex Numbers 91577 (3.5) |  |
| Differential Calculus 91578 (3.6) |  |
| Integral Calculus.91579 (3.7) |  |

**91577: Apply the algebra of Complex Numbers.**

You are advised to spend 60 minutes answering the questions in this booklet.

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**QUESTION ONE**

(a) Solve the following equation to find an expression for ***x*** in terms of ***p***:

***.***

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(b) Write in the form  where a and b are rational numbers.

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(c) Solve for *x* in terms of *k*: .

**You must show all your working steps. It is not necessary to check the validity of your answer.**

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(d) Find the roots of the equation ***z3 =*** – ***k6*** in polar form,

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in terms of ***k***, where ***k*** is a real number.

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(e) One root of the equation ***z3 + Az2 – 2z + B = 0*** is ***z = 3 + i.***

Find the value of the real numbers A and B and the other two roots.

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**QUESTION TWO**

(a) *u* and *v* are complex numbers where  and 

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Find:

(i) 4*u* + expressing your answer in the rectangular form.

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(ii)  expressing your answer in polar form .

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(b)One root of the equation , is **four** times the other root.

Find the value of **c** in terms of **b.**

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(c)

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(d) ***The complex number w = z – 4i where z = x + yi***

***z – 2***

***If w is completely imaginary, show that (x – 1)2 + (y – 2)2 = 5***

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**QUESTION THREE**

(a) Solve the equation for ***x***in terms of***p***: ***3(x – p) = 2(x + p)***

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(b) Solve for ***x*** *in terms of* ***t***:

***log(x + 4) – log(x) = log(t)***

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(c)

Express ***√2 + i√2*** in **polar** form and hence find ***(√2 + i√2)6*** in the **rectangular**

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form ***a + bi*** where ***a*** and ***b*** are rational numbers.

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(d) Find all the solutions of .

Write your solutions in rectangular form ***a + bi.***

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(e) Given ***z = x + iy*** show that if  ***│ z +1 – i │ = │(1 – i) z│***

then ***(x – 1)2 + (y + 1)2 = 4***

Assessor’s

use only

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**91578 Apply differentiation methods in solving problems**

You are advised to spend 60 minutes answering the questions in this booklet.

Assessor’s

use only

Show all working.

**QUESTION ONE**

(a) Differentiate **. You do not need to simplify your answer.**

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(b) Find the equation of the tangent to the curve **** at the point **.**

**Show any derivatives that you need to find when solving this problem.**

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(c) A volume of a weather balloon is increasing at a rate of 0.2 m3/min as it rises into the air.

Find the rate at which the radius is increasing when the radius is 0.5m.

**Show any derivatives that you need to find when solving this problem.**

Volume of a sphere = 4πr3

3

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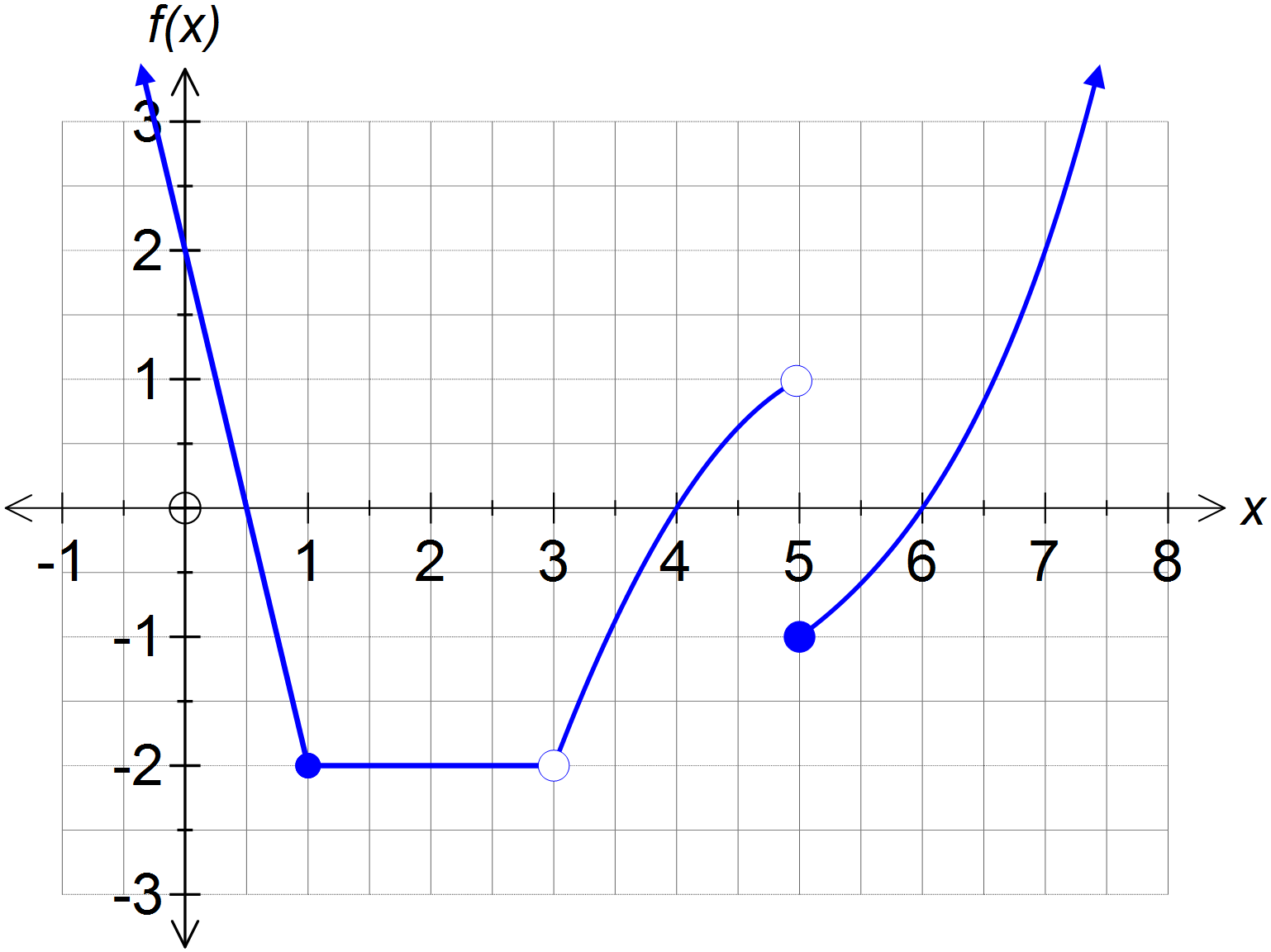
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(d) The graph below defines the function .

Assessor’s

use only



For the function  find:

(i) .

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(ii) .

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(iii) all values of *x* where is not differentiable.

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(iv) all values of *x* where *f ꞌ(x)* = 0.

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(v) all values of *x* where .

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(e) Show that the largest rectangle that can fit inside this semicircle of radius ***r***, has an area of ***r2***.

Assessor’s

use only

***y***

***x***

**Show any derivatives that you need to find when solving this problem.**

(You may assume any 2nd derivative is < 0)

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**QUESTION TWO**

(a) (a) Differentiate . **You do not need to simplify your answer.**

Assessor’s

use only

Assessor’s

use only

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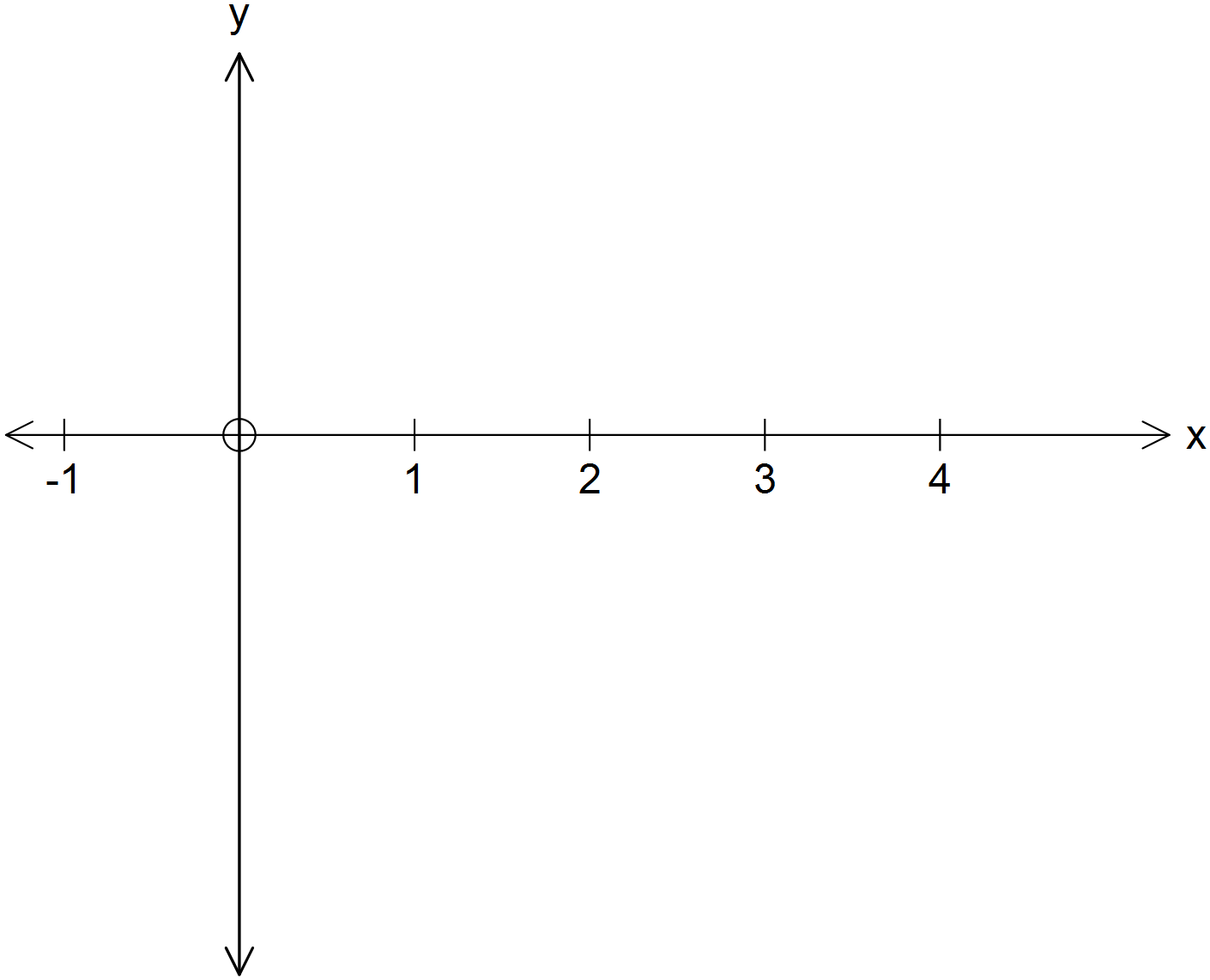
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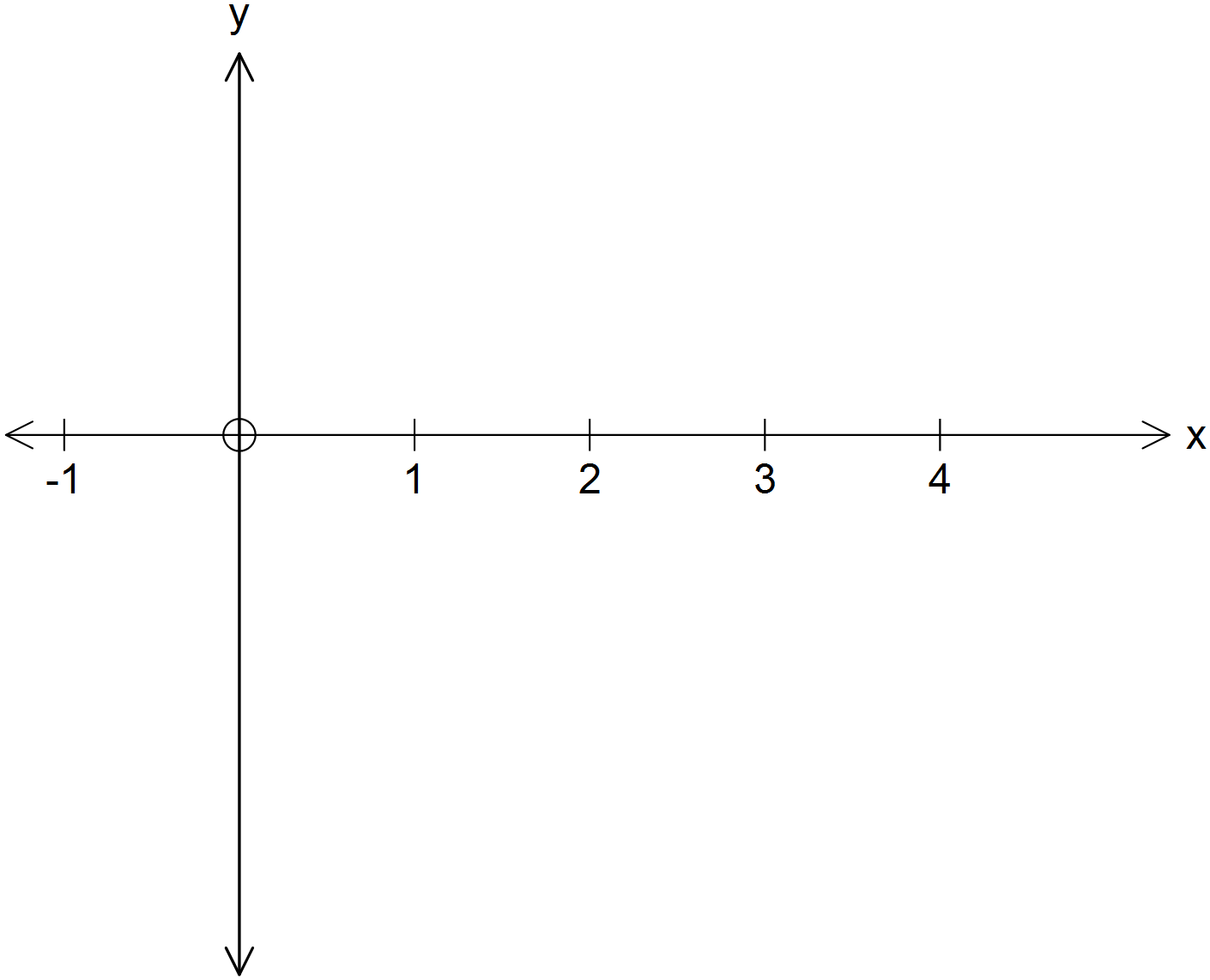
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(b) The graph shows the function .



On the axes below, sketch the graph of the **derived function** ***y*** = ***f ꞌ(x).***

***f ꞌ(x)*** 



(c) A curve is defined by the parametric equations **** and.

Assessor’s

use only

Find the **gradient** of the tangent to the curve at.

**Show any derivatives that you need to find when solving this problem.**

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(d) Show that if  then .

**Show any derivatives that you need to find when solving this problem.**

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(e) It is proposed to lay a buried water pipe from the well, marked W, on the diagram below,

Assessor’s

use only

to the Native village, marked V.

W

Native Bush

500m

P Q

ROAD

**V** 800m

The distance VQ is 800m and the well is 500m from Q.

VQ is perpendicular to QW.

The cost of laying the buried pipeline alongside the road is C dollars/metre but the cost of

laying it through the native bush area to the well is 4 times as much.

The cheapest route for the pipeline goes along the road from V to a position P where

it leaves the road and goes across the native bush.

Calculate the position of P.

(You may assume any 2nd derivative is > 0)

**Give any derivative(s) you need to find when solving this problem.**

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**QUESTION THREE**

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(a) Differentiate .

**You do not need to simplify your answer.**

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(b) Find the **equation** of the **normal** to the curve at the point where ***x = 0***

**Show any derivatives that you need to find when solving this problem.**

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(c) Find the gradient of the curve ***y4 + y = x3 – x – 4*** at the point (2, 1)

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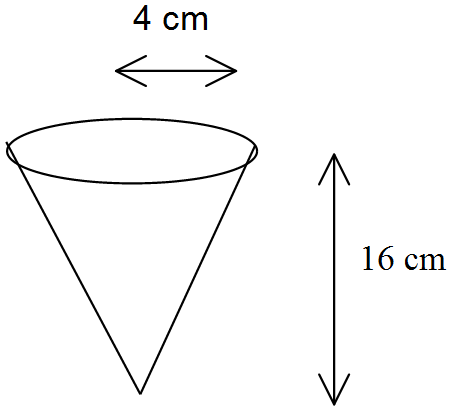
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(d) A liquid is draining through a conical filter that is

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16 cm high and with a radius of 4 cm at the top.

The liquid is coming out of the cone at a constant rate



of 2 cm3/sec.

At what rate is the depth of liquid in the cone changing

when the liquid is 8 cm deep?

Volume of a cone =

**Show any derivatives that you need to find when solving this problem.**

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(d) Find the interval in which the curve ***y = ex(x2 + 1)*** is **concave down**.

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**Show any derivatives that you need to find when solving this problem.**

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## 91579 Apply integration methods in solving problems

You are advised to spend 60 minutes answering the questions in this booklet.

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#### **QUESTION ONE**

# (a) Find the integrals:

# You do not need to simplify your answers. Do not forget the constant of integration.

(i) .

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(ii) **.**

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(b) Evaluate this indefinite integral

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**Give the results of any integration needed to solve this problem.**

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(c) A particle moves in a line from point P with an **initial** **velocity** of ***u*** m/s

and with an **acceleration** of ***a*** m/s2.

Show that the **distance** ***x***, of the particle from P at a **time** ***t*** sec is given by

***x = ut + ½ at2***

**Give the results of any integration needed to solve this problem.**

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(d) Find the area enclosed between the graphs ***y = x(x – 1)(x – 3)*** and ***y = x(x – 3)***

Assessor’s

use only

***y***



***x***

**Give the results of any integration needed to solve this problem.**

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**QUESTION TWO**

Assessor’s

use only

# (a) Find the indefinite integral

# You do not need to simplify your answer. Do not forget the constant of integration.

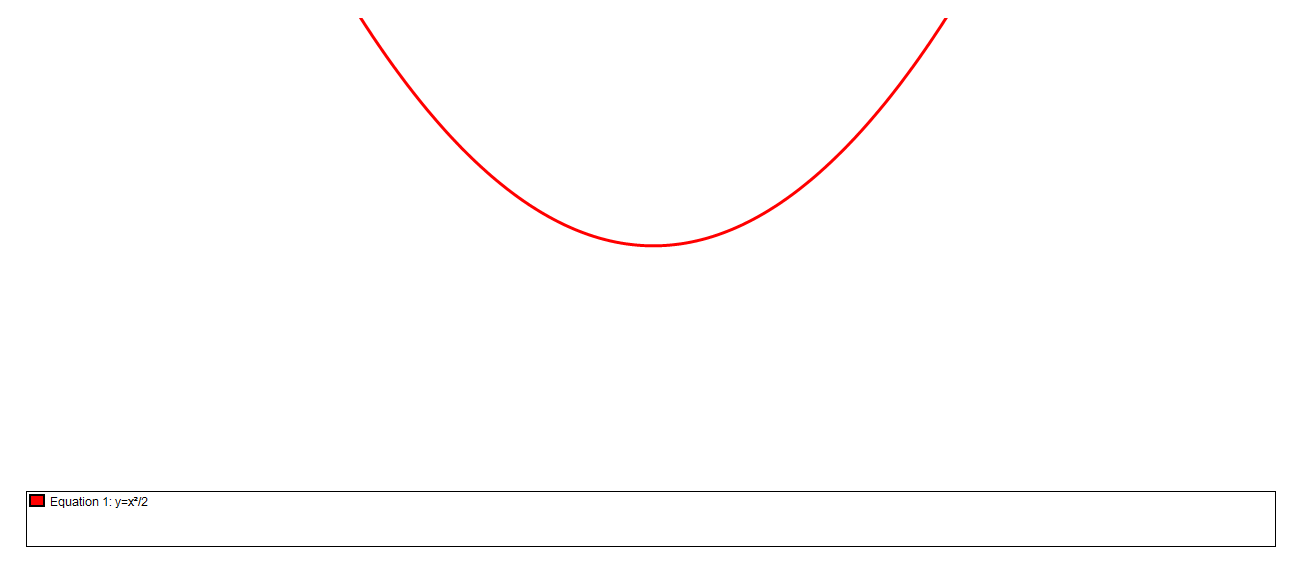
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(b) Find the area under the graph ***y = 5x4*** from ***x = k*** to ***x = 2k where k > 0***



***k 2k***

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(c) Find the integral:.

Assessor’s

use only

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(d) Given that find the value of .

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(e) When an object falls towards the earth it accelerates downwards with a constant

of 9.8 m.s – 2 due to gravity.

Assessor’s

use only

However, if we take into account air resistance the acceleration of the object is reduced and

may be expressed as:

A skydiver steps out of an aeroplane and she accelerates downwards until she reaches a

certain maximum speed. This speed is called the ***terminal velocity.***

Find an expression for her velocity at ***t*** seconds after she leaves the aeroplane, and determine

the value of her ***terminal velocity***.

**Give the results of any integration needed to solve this problem.**

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**QUESTION THREE**

Assessor’s

use only

# (a) Find the integrals:

# You do not need to simplify your answers. Do not forget the constant of integration.

(i).

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(ii) **(*sec2 4x + cosec2 2x*) *dx***

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(b) Find the particular solution of the differential equation

**** given that ***x = 1*** when ***y = 2***

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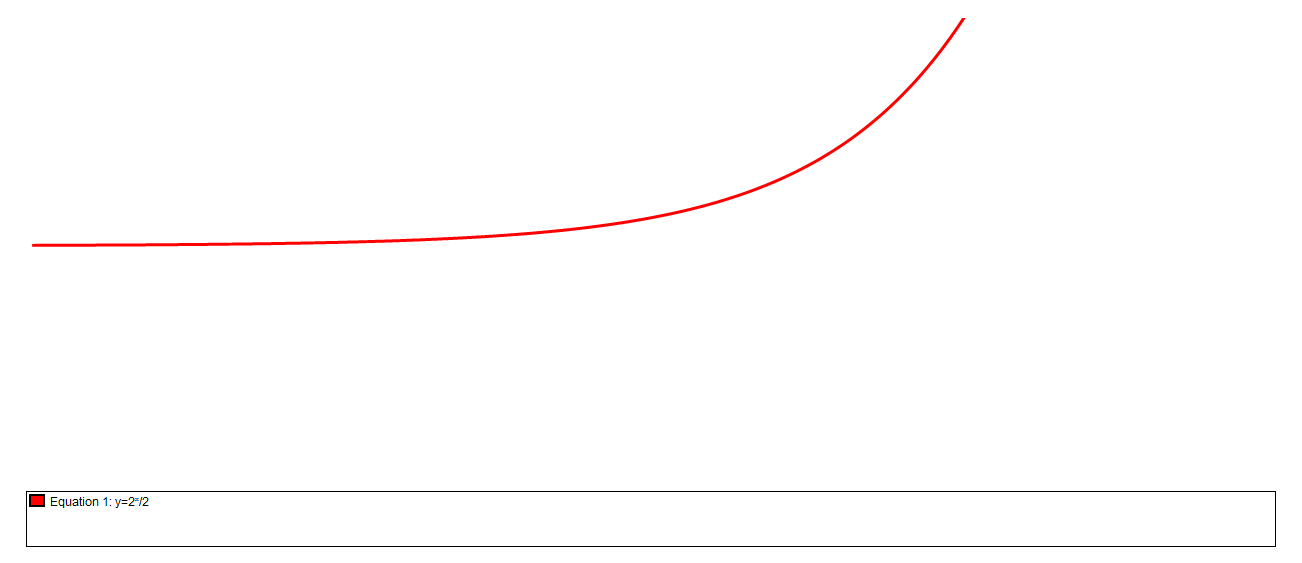
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(c)

Assessor’s

use only

The graph shown has the equation ***y = ex***



***k 4***

If the area under the curve from ***x = 0*** to ***x = k*** is a half of the area from ***x = 0*** to ***x = 4***

find the value of ***k***.

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(d) Use the substitution to show that the integral

has the exact value of

**Give the results of any integration needed to solve this problem.**

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