**ADVANCED PROBLEM ON THE REMAINDER THEOREM.**

Suppose *f(x) = x3 + x2 + 4x + 12*

Dividing *f(x) by (x – 1)* we get:

 *x2 + 2x + 6*

*x – 1 x3 + x2 + 4x + 12*

 *x3 – x2*

 *2x2 + 4x*

 *2x2 – 2x*

 *6x + 12*

 *6x – 6*

 *18* = remainder

 So *f(x) = x3 + x2 + 4x + 12 = (x – 1)(x2 + 2x + 6)* + 18

Notice that *f(1) = 1 + 1 + 4 + 12 = ( 0 )(x2 + 2x + 6)* + 18

This is the basic remainder theorem showing that the remainder is f(1)

Dividing *f(x)* by *(x – 2)* we get:

 *x2 + 3x + 10*

*x – 2 x3 + x2 + 4x + 12*

 *x3 – 2x2*

 *3x2 + 4x*

 *3x2 – 6x*

 *10x + 12*

 *10x – 20*

 *32* = remainder

 So *f(x) = x3 + x2 + 4x + 12 = (x – 2)(x2 + 3x + 10)* + 32

Notice that *f(2) = 8 + 4 + 8 + 12 = ( 0 )(x2 + 3x + 10)* + 32

This is also the basic remainder theorem showing that the remainder is f(2)

**However, if we divide by the product *(x – 1)(x – 2) = x2 – 3x + 2***

**the remainder is not just a NUMBER.**

Dividing *f(x)* by *(x – 1)(x – 2)* we get:

 *x + 4*

*x2 – 3x + 2 x3 + x2 + 4x + 12*

 *x3 – 3x2 + 2x*

 *4x2 + 2x + 12*

 *4x2 – 12x + 8*

***14x + 4* = remainder (ie it is of the form *ax + b*)**

 So *f(x) = x3 + x2 + 4x + 12 = (x2 – 3x + 2 )(x + 4) +* ***14x + 4***

 *Or f(x) = x3 + x2 + 4x + 12 = (x – 1)(x – 2)(x + 4) +* ***14x + 4***

Notice that f(1) = 1 + 1 + 4 + 12 = ( 0 ) + 14 + 4

ie the remainder when *f(x)* is divided by *(x – 1)* is still *f(1)* = 18

AND

Notice that f(2) = 8 + 4 + 8 + 12 = ( 0 ) + 28 + 4

ie the remainder when *f(x)* is divided by *(x – 2)* is still *f(2)* = 32

We can use these principles to do the following type of question:

**If *f(x)* is divided by *(x – 1)* then the remainder is 18.**

**If *f(x)* is divided by *(x – 2)* then the remainder is 32.**

**Find the remainder if *f(x)* is divided by *(x2 – 3x + 2)***

**Notice that we do not even have to know what the function *f(x*) is!**

**SOLUTION:**

The remainder will be of the form ***ax + b***

So *f(x) = (x2 – 3x + 2 )(something) +* ***ax + b***

or *f(x) = (x – 1)(x – 2)(something) +* ***ax + b***

now f(1) = 18

so subs *x* = 1 we get ***1a + b = 18***

also f(2) = 32

so subs *x* = 2 we get ***2a + b = 32***

subtracting we get ***a = 14***

 and ***b = 4***

so the remainder is ***14x + 4***

***ANOTHER PROBLEM ON REMAINDER THEOREM.***

***NOTES : 1. Divide x2 + 5x + 9 by (x – 2)***

 ***x + 7***

 ***x – 2 x2 + 5x + 9***

 ***x2 – 2x***

 ***7x + 9***

 ***7x – 14***

 ***23***

***This can be written as :***

***x2 + 5x + 9 = ( x + 7) + 23***

 ***x – 2 (x – 2)***

***or we can rearrange this as :***

***x2 + 5x + 9 = (x – 2)( x + 9) + 23***

***In general f(x) = g(x) + R***

 ***(x – 2) (x – 2)***

***Or f(x) = (x – 2)g(x) + R***

***So f(2) = 0 + R***

***2. If we divide by a quadratic expression, the remainder is not usually just a number.***

 ***x + 1***

***x2 + x + 1 x3 + 2x2 + 8x + 6***

 ***x2  + x2 + x***

 ***x2 + 7x + 6***

 ***x2 + x + 1***

 ***6x + 5***

***Remainder is 6x + 5***

***So in general, if we divide by a quadratic expression, the remainder is of the form ax + b***

 ***EXPERT QUESTION :***

 ***If an unknown function f(x) is divided by (x – 1),the remainder is 15.***

 ***If the same function is divided by (x – 2) the remainder is 28.***

***Find the remainder if f(x) is divided by (x – 1)(x – 2)***

***The remainder will be of the form ax + b***

***So f(x) = (x – 1)(x – 2) g(x) + ax + b where f(x) and g(x) are both unknown.***

***By the Remainder Thm f(1) = 15***

 ***So a + b = 15***

 ***And f(2) = 28***

 ***So 2a + b = 28***

***Producing a = 13 and b = 2***

***Remainder R = 13x + 2***