**ASSESSMENT SCHEDULE**

####  **91577 Apply the algebra of COMPLEX NUMBERS in solving problems**

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| **Achievement**  | **Achievement with Merit**  | **Achievement with Excellence**  |
|  *Apply the algebra of complex numbers in solving problems* involves:• selecting and using methods • demonstrating knowledge of concepts  and terms • communicating using appropriate  representations. |  *Apply the algebra of complex numbers, using relational thinking*, *in solving problems* involves one or more of: • selecting and carrying out a logical  sequence of steps • connecting different concepts or  representations • demonstrating understanding of  concepts • forming and using a model;and also relating findings to a context, or communicating thinking using appropriate mathematical statements. |  *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems* involves one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or  proof • forming a generalisation;and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing **limited knowledge of algebra of complex numbers in solving problems.**

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

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|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.*  |
| 1a |   | Correct expression.  |  |  |
| 1b |   | Correct solution.  |  |  |
| 1c |   k – 1 = 2√x(k – 1)2 = 4x  x = (k – 1)2/4 |  | Correct solution.  |  |
| 1d |  ***Let z rcisθ***  ***r3cis 3θ= k6cis(180 + 360n)*** ***r3 = k6 3θ=180 + 360n*** ***r = k2 θ = 60 + 120n******z1 = k2 cis 600******z2 = k2  cis 1800******z3 = k2 cis 3000*** |  Award u if changed correctly into polar form:r3cis 3θ= k6cis(180+360n)   | Correct solution either using degrees or radians. |  |
| 1e | ***If z = 3 + i then so is z = 3 – i*** ***Let the 3rd root (which is real) be p******The equation could be written as:******(z – p)( z2 – 6z + 10) = 0*** ***(using sum of roots = -(3 + i + 3 – i) = -6******and product of roots = (3 + i)(3 – i) = 10)******expanding and comparing:******z3 – (p + 6)z2 + (10 + 6p)z – 10p = 0******z3 + Az2 – 2z + B = 0******the only one to use is the coeff of z:*** ***10 + 6p = -2 so p = -2******The REAL solution is p = -2******Equating coeff of z2: A = -(6 – 2) so A = -4******Equating constant term: B = -10p = 20******Summary: solutions 3 ±i and -2*** ***A = -4 and B = 20*** | Award u if***(z – p)( z2 – 6z+ 10)***is obtained | Award r if student obtains A = -4  | Correct solution showing correct logical steps with correct mathematical statements required for t.   |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.*  |
| 2a(i) | 11 + 10i  | Correct expression.  |  |  |
| 2a(ii) | ***u + v = 5 + 5i = 5√2(cis450) or***  ***= √50(cis450)*** | Correct or in rads |  |  |
| 2b | *If and  are the two roots then* *x2 – bx + c = (x – α)( x – 4α)* *= x – 5α + 4α2**b = 5α c = 4α2**α= b c = 4 b2* *5 25*  | Award u if b = 5α c = 4α2 | Correct solution.  |  |
| 2c | ***z = ( b – bi)×(1 – i)*** ***(1 + i) (1 – i)******= (0 – 2bi) = – bi***  ***2*** ***Arg z = - π or - 900*** ***2***  | Award u for mult by ***(1 – i)***  ***(1 – i)***correctly. |  Accept 2700Correct solution.  |  |
| 2d |  ***z – 4i = x + (y – 4)i × (x – 2) – yi***  ***z – 2 (x – 2) + yi (x – 2) – yi*** ***= x(x – 2) + y(y – 4) + i ( (x – 2)(y – 4)- xy*** ***(x – 2)2 + y2*** ***Real = 0 so x2 – 2x + y2 – 4y = 0******Completing square******x2 – 2x + 1 + y2 – 4x + 4 = 1 + 4******(x – 1)2 + (y – 2)2 = 1 + 4 = 5***  |  Correct line 1  for **u**  | Correct line 3.for **r** | Correct solution showing correct logical steps with correct mathematical statements. Exclusion ***x≠2,y≠0*** not required for **t** |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.*  |
| 3a |  ***(x – p)log3 = (x + p)log2******xlog3 – plog3 = xlog2 + plog2******x(log3 – log2) = plog2 + plog3*** ***x = plog2 + plog3*** ***(log3 – log2)***  | Correct expression.  |  |  |
| 3b |  ***x + 4 = t*** ***x***  ***x + 4 = xt*** ***4 = xt – x***  ***4 = x(t – 1)***  ***4 = x*** ***(t – 1)***  | Correct solution. Restriction on t not required.  |  |  |
| 3c | ***√2 + i√2 = 2cis450*** ***So ( 2cis45 )6 = 26 cis 2700*** ***=26(cos270 + isin270)*** ***= -26i or -64i***  | Correct polar form for u | Correct solution for r |  |
| 3d |  ***r2cis2θ = 4cis(600 + 360n)******r2 = 4 2θ = 60 + 360n******r = 2 θ = 30 + 180n = 30, 210******z1 = 2cis30 = 2(cos30 + isin30)*** ***= √3 + i*** ***z2 = 2cis210 =2(cos210 + isin210)*** ***= -√3 – i***  |  Correct 1st line for u  | Correct solution.for r |  |
| 3e |  ***│ z +1 – i │ = │(1 – i) z│*** ***│ x + yi +1– i │ = │(1 – i) ( x + yi)│*** ***│(x +1) + (y – 1)i │ = │ (x + y) + ( y – x)i │*** ***(x +1)2 + (y – 1)2 = (x + y)2 + ( y – x)2*** ***x2+2x+1+y2–2y+1 = x2+2xy+y2+y2+x2–2xy*** ***2 = x2 – 2x + y2 + 2y******2 + 1 + 1 = x2 – 2x + 1 + y2 + 2y + 1*** ***4 = (x – 1)2 + (y + 1)2*** | Correct 2nd line for u | Correct 4th line for r | Correct solution showing correct logical steps with correct mathematical statements.  |

 **ASSESSMENT SCHEDULE**

####  **91578 Apply DIFFERENTIATION methods in solving problems**

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| **Achievement**  | **Achievement with Merit**  | **Achievement with Excellence**  |
|  *Apply differentiation methods in solving problems* involves*:*• selecting and using methods • demonstrating knowledge of concepts  and terms • communicating using appropriate  representations. |  *Apply differentiation methods, using relational thinking*, *in solving problems* involves one or more of: • selecting and carrying out a logical  sequence of steps • connecting different concepts or  representations • demonstrating understanding of  concepts • forming and using a model;and also relating findings to a context, or communicating thinking using appropriate mathematical statements. |  *Apply differentiation methods, using extended abstract thinking, in solving problems* involves one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or  proof • forming a generalisation;and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question demonstrating limited knowledge of differentiation techniques

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.*  |
| 1a |  | Correct derivative.  |  |  |
| 1b |  . At *x* = 0 Equation of tangent sub (0, 2)c= 2 so tan equ is  | Correct derivative with correct solution.  |  |  |
| 1c |     =0.06366  | Correct chain for for u..  | Correct derivative with correct solution.  |  |
| 1d | ***(i)*** ***(ii)  does not exist*** ***(iii) is not differentiable at*** ***(iv) 1 < x < 3******(v) x > 5*** | Correct answer for 2 situations for u   | Correct answer for at least 3 situations for r  |  |
| 1e |   (x, y)  r y x x***A = 2xy but x2 + y2 = r2******So A = 2x (r2 – x2) ½*** ***dA = 2x ½ (r2 – x2) – ½(-2x)+2(r2 – x2) ½  = 0******dx for max A*** ***2 (r2 – x2) ½  = 2x2*** ***(r2 – x2) ½***  ***r2 – x2 = x2*** ***r2 = 2x2*** ***x = r and y = r***  ***√2 √2*** ***Max A = 2xy = 2 r × r = r2*** ***√2 √2***  | Correct .  | Correct maximum for *x*. | Correct solution.  |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.*  |
| 2a |  | Correct derivative.  |  |  |
| 2b |  2 4 | Correct graph of derivative crossing x axis at x = 0, 2 and 4.  |  |  |
| 2c |   At  Grad of tangent =  | Correct .  | Correct derivative with correct solution.  |  |
| 2d |     |  Correct . | Correct derivative with correct solution.  |  |
| 2e |   ***r 500***  ***800 – x x******Total T = c(800 – x) + (5002 + x2) ½ 4c******dT = c – 1 + ½ 4×(2x) = 0 for min*** ***dx (5002 + x2) ½***  ***4x = 1******(5002 + x2) ½***  ***4x = (5002 + x2) ½***  ***16x2 = 5002 + x2***  ***15x2 = 5002*** ***x2 = 5002  x ≈ 129 m*** ***15*** | Correct equ for cost = u  | Correct derivative for r | Correct derivatives with correct solution.  |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.*  |
| 3a |  | Correct derivative.  |  |  |
| 3b | ***yꞌ = ½ ex/2 = ½ when x = 0******grad of normal = - 2*** ***if x = 0, y = 1******equ of normal is y = -2x + 1*** | Correct derivative with correct solution.  |  |  |
| 3c | ***4y3 yꞌ + yꞌ = 3x2 – 1***  ***yꞌ(4y3 + 1) = 3x2 – 1***  ***yꞌ = 3x2 – 1 = 11*** ***4y3 + 1 5*** | Correct 1st line for u | Correct solution.  |  |
| 3d |  ***ie*** ***and*** ***At h = 8  [ = 0.159]***  |   Correct  | Correct derivative with correct solution.  |  |
| 3d |   ***y = exx2 + ex*** ***yꞌ= ex2x + exx2 + ex*** ***= ex(x2 + 2x + 1)*** ***= ex(x + 1)2 = 0 at max/min/infl*** ***So x = -1 since ex never equals zero.******1st derive test for x = -1***

|  |  |  |  |
| --- | --- | --- | --- |
| ***x*** | ***-2*** | ***-1*** | ***0*** |
| ***yꞌ*** | ***+*** | ***0*** | ***+*** |

***Stationary infl pt at x = -1, y = 2e – 1*** ***yꞌꞌ = ex2 + ex2x + ex2x + exx2  + ex*** ***= ex(2 + 2 + 2x + x2 + 1)*** ***= ex(x2 + 4x + 3)*** ***= ex (x + 1)(x + 3) = 0 at infl pts******1st derive test for x = -3, y = 10e – 3***

|  |  |  |  |
| --- | --- | --- | --- |
| ***x*** | ***-4*** | ***-3*** | ***-2*** |
| ***yꞌ*** | ***+*** | ***0*** | ***+*** |

 ***y***  ***-3 -1 x******Curve is concave down for -3 < x < -1*** | Correct ***yꞌ for u*** | *Correct* ***yꞌꞌ for r*** | Correct derivatives with correct solution.  |

**ASSESSMENT SCHEDULE**

####  **91579 Apply integration methods in solving problems**

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| **Achievement**  | **Achievement with Merit**  | **Achievement with Excellence**  |
|  *Apply integration methods in solving problems* involves*:*• selecting and using methods • demonstrating knowledge of concepts  and terms • communicating using appropriate  representations. |  *Apply integration methods, using relational thinking*, *in solving problems* involves one or more of: • selecting and carrying out a logical  sequence of steps • connecting different concepts or  representations • demonstrating understanding of  concepts • forming and using a model;and also relating findings to a context, or communicating thinking using appropriate mathematical statements. |  *Apply integration methods, using extended abstract thinking, in solving problems* involves one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or  proof • forming a generalisation;and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing limited knowledge of integration techniques.

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |
| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.*  |
| 1a(i) | ***6e2x + c or 3e2x  + c*** ***2*** | Correct integration.  |  |  |
| 1a(ii) |  dx = *3x4 + ln│x│ + c* | Correct integration.  not essential. |  |  |
| 1b | ***u = x – 2 so du = dx and x = u + 2*** dx = =  | Correct substitution line 2 for u | Correct integration with correct solution.  |  |
| 1c | ***dv = a******dt******so v = at + c subs v = u, t = 0 so c = u******v = dx = u + at*** ***dt******x = ut + ½ at2 + d subs x = 0, t = 0*** ***so d = 0******hence x = ut + ½ at2***  | 2 Correct integrations.  | CorrectIntegrations with both constants c and d evaluated. |  |
| 1d |  Intersection points: x(x – 1)(x – 3) = x(x – 3)BAx(x – 1)(x – 3) – x(x – 3) = 0 x(x – 3) ( x – 1 – 1) = 0 x(x – 3)(x – 2) = 0intersection points x = 0, 2 and 3 Area A = cubic – parabola = – 4x2 + 3x – (x2 – 3x) dx = x4 – 4x3 + 3x2 – (x3 – 3x2) 4 3 2 3 2 = 2.6667 by calc or 8 3Area B = parabola – cubic  = (x2 – 3x) – (x3 – 4x2 + 3x) dx = (x3 – 3x2) – ( x4 – 4x3 + 3x2 )  3 2 4 3 2  = 0.4165 by calc or 5 12 Total area between curves = 3.083 or 37 12 | Correct intersection for u | Correct idea of area = area under cubic – area under parabola (and vice versa) shown with equations.  | Correct solution showing correct integrations and correct mathematical logic and statements.If middle intersection is only “assumed” to be x = 2 only award an ***r*** |
| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.*  |
| 2a | ***sin 3x + cos 5x + c*** ***3 5*** | Correct integration.  |  |  |
| 2b | A =  = ***31k5***   | Correct integration. (accept 32k5 – k5) |  |  |
| 2c |   =  | Correct integration with minor error in division (or substitution).  not essential. | Correct integration with correct solution.  |  |
| 2d | Area   ***63a = 315*** ***a = 5***  | Correct integration.  | Correct integration with correct solution.  |  |
| 2e |  ***dv = 9.8e – 0.175t*** ***dt******v = - 9.8 e – 0.175t + c***  ***0.175******Subs t = 0, v = 0 so 0 = -56 + c******v = 56 – 56e – 0.175t******as t increases e – 0.175t 0******so terminal velocity is 56 m/s*** | Correct integralfor u | Correct equation v = 56 – 56e – 0.175tfor r | Correct solution showing correct integrations and correct terminal vel with clear mathematical statements.  |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.*  |
| 3a(i) | .= 3sin4x + c | Correct integration.  |  |  |
| 3a(ii) | ***tan 4x – cot 2x + c*** ***4 2***  | Correct integration.  |  |  |
| 3b | ***y =ex – ln │x│ + c******subs x = 1, y = 2******2 = e – 0 + c so c = 2 – e******y = ex – ln│ x│ + 2 – e***  |  Correct integration.  | ││ not essentialCorrect integration with correct constants |  |
| 3c |  ***Large area = e4 – 1*** ***Small area = ek – 1*** ***2(ek – 1) = e4 – 1*** ***2ek – 2 = e4 – 1******2ek = e4 + 1*** ***ek = e4 + 1 = 27.799*** ***2*** ***k = 3.325*** |  2 Correct definite  integrations. | Correct integrations with correct calculation of k. |  |
| 3d | ***x = sint so dx = cost dt******√(1 – x2) = √(1 – sin2t) = cost******If x = 1 , t = π*** ***2***  = ***Use cos 2t = 2 cos2t – 1*** ***So cos2t = ½ ( cos 2t + 1)*** π/2= ½ sin 2t + t 2 0= 1 × π = π 2 2 4 | Reducing integral to cos2t for u | Using cos 2t formula for r | Correct integration with correct logical steps and correct solution. |