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As part of his summer job at a restaurant, Jim learned to cook up a big pot of soup late at night, just before closing time, so that there would be plenty of soup to feed customers the next day.

He also found out that, while refrigeration was essential to preserve the soup overnight, the soup was too hot to be put directly into the fridge when it was ready.

(The soup had just boiled at 100 ^{0}C, and the fridge was not powerful enough to accommodate a big pot of soup if it were any warmer than 20 ^{0}C).

Jim discovered that by cooling the pot in a sink full of cold water, (kept running, so that its temperature was constant at 10 ^{0}C) and stirring occasionally, he could bring the temperature of the soup down to 60 ^{0} C in 10 minutes.

How long before closing time should the soup be ready so that Jim could put it in the fridge and leave on time?

Newton's Law of Cooling says that the rate at which a body cools is proportional to the difference between its temperature and its surroundings.

Let T = temperature of the soup and $\mathbf{T_0}$ = the temperature of the surroundings.

The difference $D = T - T_0$

So $\frac{dD}{dt} = kD$ producing $\int \frac{dD}{D} = \int k\,dt$ so $\ln(D) = kt + c$

at $t = 0$, $T = 100^0C$, $T_0 = 10^0C$ so $D = 90^0C$

subs $\ln(90) = 0 + c$

so $\ln(D) = kt + \ln(90)$

$\ln(D) - \ln(90) = kt$

$\ln\left(\frac{D}{90}\right) = kt$

At $t = 10$, $T = 60^0C$ so $D = 50^0C$

Subs $\ln\left(\frac{50}{90}\right) = 10k$ so $k = -0.058779$

$\ln\left(\frac{D}{90}\right) = -0.058779t$

$\frac{D}{90} = e^{-0.058779t}$ so $D = 90e^{-0.058779t}$

To find when T has reduced to 20^0C we subs $D = 10$

in $\ln\left(\frac{D}{90}\right) = -0.058779t$ so $\ln\left(\frac{10}{90}\right) = -0.058779t$

Producing $t = 37.4$ min

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