**QUADRATIC THEORY IN BRIEF.**

Given *ax2 + bx + c = 0*

*then x = − b ± √* ***(b2 − 4ac****)*

*2a*

1. Use the quadratic formula to solve these equations (solutions to 2 dec pl.)

(a) 3x2 + 9x + 5 = 0

*ax2 + bx + c = 0*

*x = − b ± √* ***(b2 − 4ac****)*

*2a*

*= − 9 ± √* ***(92 − 4×3×5****)*

*2×3*

*= − 9 ± √* ***(21****) (NB this is the “exact” answer!)*

*6*

***x = –0.74 or –2.26 (NB this answer is only an approximation)***

(b) 5x2 − 7x − 11 = 0

*ax2 + bx + c = 0 a = 5 but b = -7 and c = - 11*

*x = − b ± √* ***(b2 − 4ac****)*

*2a*

*= +7 ± √* ***(49− 4×5×(-11)****)*

*2×5*

*= +7 ± √* ***(49− 4×5×(-11)****)*

*2×5*

*= +7 ± √* ***(269)*** *(NB this is the “exact” answer!)*

*10*

***x = 2.34 or –0.94 (NB this answer is only an approximation)***

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*x = − b ± √* ***(b2 − 4ac****)*

*2a*

THE DISCRIMINANT **Δ** = ***b2 – 4ac***

***REMEMBER: The solutions of an equation are where the***

***graph of the equation crosses the x axis.***

**“COMPLETING THE SQUARE” METHOD.**

2. Show clearly how to solve each of the following 4 equations by completing the square (even though 2 of them factorise)

and state how the **discriminant** affects the type of solutions.

(a) x2 – 8x + 7 = 0

***x2 – 8x = –7 y = x2 – 8x + 7***

***x2 – 8x + 16 = –7 + 16 =(x – 1)(x – 7)***

***(x – 4)2 = 9***

***so x – 4 = 3 or x – 4 = –3 1 7***

***x = 7 or 1***

***The solutions are where the graph crosses the x axis.***

***Δ = 82 – 4×1×7***

***= 64 – 28***

***= 36 (a perfect square) so we get 2 rational sols***

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(b) x2 – 8x + 16 = 0 ***y = x2 – 8x + 16***

***x2 – 8x = -16 = (x – 4)2***

***x2 – 8x + 16 = – 16 + 16***

***(x – 4)2 = 0***

***x = 4***

***The solutions are where the graph crosses the x axis. 4***

***Δ = 82 – 4×1×16***

***= 64 – 64***

***= 0 so we get 1 rational sol.***

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(c) x2 – 8x + 5 = 0

***x2 – 8x = -5 y = x2 – 8x + 5***

***x2 – 8x + 16 = – 5 + 16***

***(x – 4)2 = 11***

***x – 4 = ±√11***

***x = 4 ±√11 1 7***

***≈ 7.32, 0.683***

***The solutions are where the graph crosses the x axis.***

***Δ = 82 – 4×1×5***

***= 64 – 20***

***= 44 so we get 2 irrational sols. because 44 does not have***

***an exact square root.***

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(d) x2 – 8x + 20 = 0

***x2 – 8x = -20 y = x2 – 8x + 20***

***x2 – 8x + 16 = – 20 + 16 = – 4***

***(x – 4)2 = – 4***

***Can’t find √– 4***

***The solutions are where the graph crosses the x axis***

***but it does not cross the x axis so there are***

***no real solutions.***

***4***

***Δ = 64 – 80 = – 16 so no real sols.***

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3. The Discriminant is ***Δ = b2 − 4ac.***

State what **type** of solutions you get if the discriminant is :

(a) 0 (b) 1 or 4 or 9 or 16 etc

***= 1 rat sol = 2 rat sol***

***(graph sits on x axis) (graph crosses x axis at***

***whole numbers or fractions)***

(c) 2 or 3 or 5 or 6 etc (d) -1 or -5 or -76 etc

***= 2 irrat sol = NO real solutions***

***(graph crosses x axis at numbers (graph does not cross x axis)***

***which are SURDS (eg √3) )***

EXAMPLES

1.

Find the **value** of ***p*** so that

***x2 – 10x + p = 0*** has one solution.

This will have only 1 solution if the

graph sits on the ***x*** axis.

In which case, the discriminant = 0

***Δ = 100 – 4p = 0***

***100 = 4p***

***25 = p***

***Note: if p = 25, the equation is x2 – 10x + 25 = 0***

***so that (x – 5)2 = 0***

***and the only solution is x = 5***

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2.

Find ***p*** so that ***x2 + (p+2)x + (3p –2) = 0***

has only one rational solution.

This will have only 1 solution if the

graph sits on the ***x*** axis.

In which case, the discriminant = 0

***Δ = (p + 2)2 – 4(3p – 2) = 0***

***p2 + 4p + 4 – 12p + 8 = 0***

***p2 – 8p + 12 = 0***

***(p – 2)(p – 6) = 0***

***p = 2 or 6***

**Some students find this “double” answer confusing:**

**It means that if p = 2 the equation *x2 + (p+2)x + (3p –2) = 0***

becomes **x2 + 4x + 4 = 0**

**and THIS equation only has 1 solution (x = -2) -2**

**AND**

**It means that if p = 6 the equation *x2 + (p+2)x + (3p –2) = 0***

becomes **x2 + 8x + 16 = 0**

**and THIS equation only has 1 solution (x = -4) -4 -2**