**Name………………………………………………….**

**MATHEMATICS WITH CALCULUS**

**2015**

**TIME ALLOWED: 3 hours**

# Show ALL working for ALL questions

**If you need more space, use the back pages of each assessment.**

**Unless otherwise stated, 3 significant figure numerical answers will be adequate.**

**Mathematical formulae and tables are supplied.**

|  |  |
| --- | --- |
| **Achievement Standard Assessment** | **Grade** |
| Complex Numbers 91577 (3.5) |  |
| Differential Calculus 91578 (3.6) |  |
| Integral Calculus.91579 (3.7) |  |

**91577: Apply the algebra of Complex Numbers.**

You are advised to spend 60 minutes answering the questions in this booklet.

Assessor’s

use only

#

**QUESTION ONE**

(a) Given that ***(x – 1)*** is a factor of  ***3cx3 – 4cx2 – 8cx +18 = 0*** find the value of ***c*** where ***c*** is real.

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 (b) If ***u = 1 + i,***  find  ***u8*** in the polar form ***rcis(θ)***

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 (c) Find any valid solutions of the equation: 

 **You must show all your working steps. It is necessary to check the validity of your answers.**

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(d) Solve the equation **z*5 = 32ni*** in terms of ***n***, where ***n*** is a real number and express

Assessor’s

use only

 your solutions in polar form.

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 (e) Find the equation of the locus described by =

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**QUESTION TWO**

(a) Solve the equation ***z2 – 2z + 25 = 0***  giving your solutions in the form ***a ± ib√c***

Assessor’s

use only

where ***a, b and c*** are positive whole numbers and ***b*** ≠ ***1***

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(b)Complex numbers ***u*** and ***v*** are represented on the Argand diagram.

 If ***w = u***  show the position of ***w*** on Argand diagram below.

 ***v***

 Im



***u***

***v***

Re

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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 (c) One root **of *v3 – 2v2 – 3v + k = 0*** is ***v = –1 + 2i***

 Find the value of ***k*** and the other two roots. (note: ***k*** is a real number)

Assessor’s

use only

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(d)(i) The complex number**z**satisfies the equation where **z *= x + yi***

Carefully draw the locus of **z** on the Argand diagram below.

 Im



Re

 (ii) What is the minimum value of **Im(z)** ?

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Assessor’s

use only

(e) If **z *= a + ib*** where ***b > a > 0*** and ***arg(z) = –* π** show that ***a = – 2b***

 ***3 + i 2***

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**QUESTION THREE**

(a) Expand (1 + √2)(3 – √2)2 and express your answer in the form ***a + b√2***

Assessor’s

use only

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(b) If ***u = 8cis(5θ)*** and ***v = 4cis(3θ)*** write ***u*** in polar form.

 ***v***

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 (c) Find the exact value of )9 expressing your answer in the form ***a + ib***

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 (d) If z = ***x + iy and w = 1 show that w = x2 – y2 – 2xyi***

Assessor’s

use only

 z**2**  ***(x2 + y2)2***

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 (e) ***a, b, c*** and ***d*** are the four roots of the quartic equation:

 ***x4 + px3 + qx2 + rx + t = 0*** where ***p, q r*** and ***t*** are real numbers

 (i) show that ***a + b + c + d = -p***  and ***abcd = t***

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(ii) Hence show that ***1 + 1 + 1 + 1 = – p***

 ***bcd acd abd abc t***

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**91578: Apply differentiation methods in solving problems**

You are advised to spend 60 minutes answering the questions in this booklet. Show all working.

Assessor’s

use only

**QUESTION ONE**

(a) Differentiate ***y = 8sin(4x)* You do not need to simplify your answer.**

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 (b) Find the gradient of the NORMAL to the curve ***y = (x3 + 2x)2***

 at the point (1, 9)

 **You must use calculus and clearly show your working, including any derivatives you need to**

 **find when solving this problem.**

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(c) If ***x = cos(2t)*** and ***y = 2sin(t)*** show that ***dy = – 1 .***

Assessor’s

use only

 ***dx sin(t)***

 **You must use calculus and clearly show your working, including any derivatives you need to**

 **find when solving this problem.**

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 (d) A curve is defined by ***y = e 8 – 4x  + 4x***

 Find the ***x*** coordinates of any points on the curve where the gradient is **zero.**

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 (e) If the slant height of a cone is 8√3 cm prove that the maximum volume

Assessor’s

use only

 is when ***h*** = 8 cm and ***r*** = 8√2 cm

Volume of a cone = ***πr2h***

 ***3***

 ***h*** 8√3

 ***r***

  **Show any derivatives that you need to find when solving this problem.**

 **(You may assume any 2nd derivative is < 0. )**

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**QUESTION TWO**

(a) (a) Differentiate ***y = ln(x2 + 1)*** **You do not need to simplify your answer.**

Assessor’s

use only

Assessor’s

use only

 ***3x + 4***

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 (b) Find the **gradient** of the tangent to the curve ***y = 3e2x – 8***  where ***x = 4***

 **Show any derivatives that you need to find when solving this problem.**

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 (c) The graph below shows the piece-wise function ***y = f(x)***

 ***y***  ***x***

-4 -3 -2 -1 0 1 2 3 4 5

(i) For what ***x*** values is ***f(x)*** not **continuous?** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(ii) For what ***x*** values is ***f(x)*** not **differentiable? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

(iii) For what ***x*** values is ***f(x)*** not **defined? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

(iv) What is ***lim f(x) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_***

 ***x⭢ 0***

(v) What is ***lim f(x) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_***

 ***x⭢ 1***

(vi) What is ***f(1)*** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Assessor’s

use only

(d) A type of fungus exists for a period of 15 days.

 The volume ***V***cm3, of the fungus at ***t*** days is given by ***V = 16 – t – 16 .***

 ***(t + 1)***

Find the maximum volume of the fungus.

 **Show any derivatives that you need to find when solving this problem.**

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 (e) The curve below has the equation ***y = x2 The units are in centimetres.***

 

The point P moves along the ***x*** axis from ***x = 0*** to ***x = 10*** at a rate of 2 cm/sec.

The point S is on the curve ***y = x2***

 ***10***

and is always vertically above P.

Find the **rate** at which the area of rectangle PQRS is changing when P is at ***x*** = 5 cm

**Give any derivative(s) you need to find when solving this problem.**

***10***

S

R

 P Q

 ***x***

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**QUESTION THREE**

Assessor’s

use only

(a) Differentiate 

 **You do not need to simplify your answer.**

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 (b) Find any ***x*** values for which the function has stationary points.

 **Show any derivatives that you need to find when solving this problem.**

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 (c) For what values of ***x*** is the function ***y = xln(x) – 2x*** decreasing.

 **You must use calculus and clearly show your working, including any derivatives you need to**

 **find when solving this problem.**

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 (d) A meteor M, which has reached a terminal velocity of 90 m/s, is falling vertically

 to the earth’s surface and will hit the ground at Q.

Assessor’s

use only

 The meteor is being filmed by a camera at C.

 The distance CQ is 800 metres.

 M

 ***θ***

 Q 800m C

 Find the rate at which the camera is rotating when the meteor is at a height of 500m.

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Assessor’s

use only

(e) ABC is an equilateral triangle with sides 20 cm long.

 PQRS is a rectangle which fits inside the triangle as shown below.

 A

 S R

 **B P Q C**

Find the dimensions and area of the largest rectangle which can fit inside triangle ∆ABC.

**(You do not have to prove your value is a maximum. You may assume any 2nd derivative < 0)**

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 91579: **Apply integration methods in solving problems**

You are advised to spend 60 minutes answering the questions in this booklet.

Assessor’s

use only

#### **QUESTION ONE**

# (a) Find the integrals:

# You do not need to simplify your answers. Do not forget the constant of integration.

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(b) Calculate

 **Give the results of any integration needed to solve this problem.**

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 (c) The velocity ***v metres/sec,*** of an object at ***t*** sec is given by the equation:

 ***v = 2t –***

Assessor’s

use only

 Find the distance moved by the object from ***t*** = 1 to ***t*** = 4

 **Give the results of any integration needed to solve this problem.**

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(d) During a chemical reaction, the rate of decrease in mass ***m***, of a substance

 is proportional to

 At the start of the reaction ***m = 8 g*** and at ***t = 2*** hours, ***m = 1 g.***

 At what time will the substance have completely gone?

 **Give the results of any integration needed to solve this problem.**

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(e) The graphs below have the equations ***y = (p2+1) – x2***and ***y = p2 where p > 1***

Assessor’s

use only

 ***x2***

 ***y***

***x***

(i) Show that the intersection points are ***x = ± 1*** and ***x = ± p***

(ii) Show that the shaded area is ***2p3 – 2p2 + 2p – 2***

 ***3 3***

 **Give the results of any integrations needed to solve this problem.**

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**QUESTION TWO**

Assessor’s

use only

# (a) Use the values given in the table below to find an approximation to using Simpson’s Rule.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| *f(x)* | 3 | 9 | 13 | 15 | 11 | 5 | 1 |

 **Give your answer correct to 3 significant figures.**

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 (b) Solve the differential equation

 given that when ***x = 1, = 4 and y = 2***

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Assessor’s

use only

 (c) The equation of the graph below is

 ***y***

 

B

A

 1 k 8 ***x***

 Find the value of k so that the areas A and B are equal.

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(d) If and ***x =* 1**when ***y =* 1**, find ***y*** when ***x = 2***

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 (e)

 Find the indefinite integral:

Assessor’s

use only

 **Give the results of any integration needed to solve this problem.**

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 **QUESTION THREE**

Assessor’s

use only

# (a) Find the integral:

# You do not need to simplify your answers. Do not forget the constant of integration.

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# (b) Find the integral:

#

#

# You do not need to simplify your answers. Do not forget the constant of integration.

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Assessor’s

use only

 (c) Evaluate the indefinite integral

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 d) Evaluate the integral

**Give the results of any integration needed to solve this problem.**

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Assessor’s

use only

(e) Radioactive waste is stored in lead containers until the radioactivity

 decays to a safe level.

 The rate of decay of this particular isotope is proportional to the amount

 of radiation the isotope already has.

 At t = 100 days, the radiation was checked and found to be 150 curies.

 At t = 200 days, the radiation was checked again and found to be 90 curies.

 The radioactive waste will be relatively safe when the radiation has

 reached 20 curies or less.

 Find how long this will take.

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