**ASSESSMENT SCHEDULE**

####  **91577 Apply the algebra of COMPLEX NUMBERS in solving problems**

|  |  |  |
| --- | --- | --- |
| **Achievement**  | **Achievement with Merit**  | **Achievement with Excellence**  |
|  *Apply the algebra of complex numbers in solving problems* involves:• selecting and using methods • demonstrating knowledge of concepts  and terms • communicating using appropriate  representations. |  *Apply the algebra of complex numbers, using relational thinking*, *in solving problems* involves one or more of: • selecting and carrying out a logical  sequence of steps • connecting different concepts or  representations • demonstrating understanding of  concepts • forming and using a model;and also relating findings to a context, or communicating thinking using appropriate mathematical statements. |  *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems* involves one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or  proof • forming a generalisation;and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing **limited knowledge of algebra of complex numbers in solving problems.**

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

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| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.*  |
| 1a | ***f(1) = 0******so 3c – 4c – 8c +18 = 0*** ***18 = 9c***  ***2 = c*** | Correct expression.  |  |  |
| 1b | ***u = 1 + i = 2 ½ cis(450)******u8 = 24cis3600***  | Correct solution. Degrees or rads |  |  |
| 1c |   ***4x – 8 = x2 – 4x + 4*** ***0 = x2 – 8x + 12*** ***0 = (x – 2)(x – 6)******x = 2 or 6******check if x = 2 check if x = 6******lhs = 0, rhs = 0 lhs = 4 rhs = 4******so x = 2 is valid so x = 4 is valid***  | Award u if minor error ( or no check done) | Award r only forCorrect solution with check.  |  |
| 1d |  ***Let z rcisθ* z*5 = 32ni*** ***r5cis 5θ= 32ncis(90 + 360n)*** ***r5 = 32n 5θ=90 + 360n*** ***r = 2n0.2 θ = 18 + 72n******z1 = 2n0.2 cis 180******z2 = 2n0.2  cis 900******z3 = 2n0.2 cis 1620******z4 = 2n0.2cis2340******z4 = 2n0.2cis3060*** |  Award u if changed correctly into polar form:r5cis 5θ= ***32ncis(90 + 360n)*** either using degrees or radians.   | Correct solution either using degrees or radians. |  |
| 1e |  = ***│(x+1) + (y + 1)i│=│(x + 2) + iy│******Squaring each modulus:******(x + 1)2 + (y + 1)2 = (x + 2)2 + y2******x2 + 2x + 1 + y2 + 2y + 1 = x2 + 4y + 4 + y2*** ***2x + 2 + 2y = 4y + 4*** ***2x – 2 = 2y*** ***y = x – 1***   | Award u if ***│(x+1) + (y + 1)i│******=│(x + 2) + iy│***is found correctly | Award r if ***(x + 1)2 + (y + 1)2 = (x + 2)2 + y2***is obtained  | Correct solution showing correct logical steps with correct mathematical statements. |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.*  |
| 2a | *z2 – 2z + 25 = 0**z2 – 2z + 1 = 1 – 25* *(z – 1)2 = -4×6* *z – 1 = ±i2√6* *z = 1 ± i2√6* | Correct expression. |  |  |
| 2b | ***u = 8 – 4i******v = 1 – 3i******w = 2 + 2i correctly placed at (2, 2)*** | If ***w*** correctly worked out and placed on Argand diagram award u |  |  |
| 2c | ***1 root of v3 – 2v2 – 3v + k = 0*** is ***–1 + 2i*** ***another is –1 – 2i (conjugate)******Let 3rd root (which must be real) = b******(z – b)(z2 + 2z + 5) = 0******Comparing terms in z2: 2 – b = -2******So b = 4 = 3rd solution*** ***Comparing number terms: -5b = k*** ***-20 = k***   | Award u if theroot ***–1*** ***– 2i*** is stated | Correct solution.  |  |
| 2d | ***(x – 2)2 + (y + 2)2 = 4******(Locus is a circle, centre (2, -2) radius 2)*** Min value of **Im(z) = -4** | Award u for correct locus drawn. | Award r if min value also stated.  |  |
| 2e | ***z = (a + bi) (3 – i) = 3a + b + i(3b – a)*** ***(3 + i) (3 – i) 10 10******3b – a = -1 if angle = -450******3a + b 3b-a*** ***-45******So 3b – a = - 3a – b 3a-b******4b = -2a so a = -2b*** | Correct expression without ***i2***for u | Arg(z) = - 450 or -1350 interpreted to give acorrect relationship such as im(z) = -1 for rre(z) | Correct solution. |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply the algebra of complex numbers in solving problems.* | *Apply the algebra of complex numbers, using relational thinking, in solving problems.* | *Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.*  |
| 3a | (1 + √2)(3 – √2)2 =(1 + √2)(11 - 6√2) = -1 + 5√2  | Correct answer required for u |  |  |
| 3b | ***u = 8cis(5θ)*** and ***v = 4cis(3θ)*** ***8cis (5θ) = 2cis(2θ)******4cis(3θ)***  | Correct solution. required for u |  |  |
| 3c | )9  1= ( 1cis(-45) )9= 1 cis(-405) = cis (-45)= )  | Change to polar form required for u | Correct solution required for r |  |
| 3d |  ***w = 1*  *= 1 × (x – iy)2*** ***z2 (x + iy)2 (x – iy)2***  ***= x2 + y2 – 2xyi*** ***(x + iy)(x – iy)(x + iy)(x + iy)***  ***= x2 + y2 – 2xyi = x2 + y2 – 2xyi***  ***(x2 + y2) (x2 + y2) (x2 + y2)2*** | Multiplied by conjugate in some form for u Could have expanded 1 . . (x + iy)2at start but method shown is better. | Correct **exact** solution for r. |  |
| 3e | ***(i) x4 + px3 + qx2 + rx + t = 0******Equ is (x – a)(x – b)(x – c)(x – d) = 0*** ***(x2 – (a+b)x + ab)(x2 – (c+d)x + cd)******Coeff of x3: p = – (a + b) – (c + d)***  ***So a + b + c + d = –p******And...******(x2 – (a+b)x + ab)(x2 – (c+d)x + cd)******Number term: t = abcd*** ***(ii)******Dividing: –p = a + b + c + d*** ***t abcd***  ***= a + b + c + d .*** ***abcd abcd abcd abcd*** ***= 1 + 1 + 1 + 1***  ***bcd acd abd abc***  | Correct 2nd line for u | Compared coefficient of ***x3*** to get 1st relationshipand number term to get 2nd. | Divided correctly and clearly to get part (ii) |

**ASSESSMENT SCHEDULE**

####  **91578 Apply DIFFERENTIATION methods in solving problems**

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| **Achievement**  | **Achievement with Merit**  | **Achievement with Excellence**  |
|  *Apply differentiation methods in solving problems* involves*:*• selecting and using methods • demonstrating knowledge of concepts  and terms • communicating using appropriate  representations. |  *Apply differentiation methods, using relational thinking*, *in solving problems* involves one or more of: • selecting and carrying out a logical  sequence of steps • connecting different concepts or  representations • demonstrating understanding of  concepts • forming and using a model;and also relating findings to a context, or communicating thinking using appropriate mathematical statements. |  *Apply differentiation methods, using extended abstract thinking, in solving problems* involves one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or  proof • forming a generalisation;and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question demonstrating limited knowledge of differentiation techniques

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) |   |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.*  |
| 1a | ***y = 8sin(4x)******yꞌ = 32cos(4x)*** | Correct derivative.  |  |  |
| 1b | ***y = (x3 + 2x)2******yꞌ = 2(x3 + 2x)(3x2+2) = 2×3×5 = 30******so gradient of normal = -1*** ***30***  | Correct answer = u  |  |  |
| 1c | ***x = cos(2t)*** and ***y = 2sin(t)******dx = –2sin(2t) dy = 2cos(t)******dt dt******dy = 2cos(t) = cos(t) = – 1 .*** ***dx –2sin(2t) –cos(t)sin(t) sin(t)*** | Correct up to ***dy = 2cos(t)*** ***dx -2sin(2t)***= u  | Correctly simplified = r  |  |
| 1d | ***y = e 8 – 4x  + 4x******yꞌ= -4e8 – 4x + 4 = 0 at stat pts******so 4 = 4e8 – 4x*** ***1 = e8 – 4x*** ***0 = 8 – 4x******So x = 2***  | ***Correct dy for u***  ***dx******and state that*** ***dx = 0*** ***dt*** | Correct answer for for r  |  |
| 1e |  If the slant height of a cone is 8√3 cm prove that the maximum volume is when ***h*** = 8 cm and ***r*** = 8√2 cm   ***r2+ h2 = 192*** ***r2= 192 – h2***  ***h*** 8√3 ***r*** ***V = π (192 – h2)h = π (192h – h3)*** ***3 3******dV = π (192 – 3h2) = 0 for max vol******dh 3******192 = 3h2  so h2 = 64*** ***h = 8 (not -8)***and ***r2= 192 – h2*** ***= 192 – 64***  ***= 128*** ***= 64×2******So r = 8√2***  | Correct Equ for V by eliminating r or h but preferably r | Correct derivative and solving for derivative = 0 | Correct r and h found for t |

| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.*  |
| 2a | ***y = ln(x2 + 1)***  ***3x + 4******yꞌ =(3x + 4)× 2x – ln(x2 + 1) ×3*** ***(x2 + 1) .*** ***(3x + 4)2*** | Correct derivative.  |  |  |
| 2b | ***y = 3e2x – 8***  where ***x = 4*** ***yꞌ = 6e2x – 8*** ***sub x = 4******yꞌ = 6e0 = 6×1 = 6*** | Correct derivative and sub ***x = 4*** to get 6for u |  |   |
| 2c |  ***(i) not cont if x = 0, 1******(ii) not diff if x = 0, 1, 4******(iii)not defined if x = 0******(iv) lim f(x) = 1*** ***x⭢ 0******(v) lim f(x) = not defined*** ***x⭢ 1******(vi) f(1) = 3***  |  3 Correct for u | 5 Correct for r |  |
| 2d | ***V = 16 – t – 16 (t + 1) – 1*** ***dV = - 1 + 16(t + 1) – 2  = 0 for max V******dt*** ***16 = 1 so (t + 1)2 = 16*** ***(t+1)2*** ***t + 1 = ±4 so t = 3 or -5 but 0* ≤ *t* ≤ 15*****subs t = 3 max V = 16 – 3 – 4 = 9cm3*** |  |  |  |
| 2e | ***dx = 2cm/sec Area = base×height******dt = (10 – x)× x2***  ***10*** ***A = x2 – x3*** ***10*** ***dA = 2x – 3x2******dx 10******dA = dA × dx = (2x – 3x2 ) × 2******dt dx dt 10******sub x = 5 , dA = (10 – 7.5) ×2 = 5 cm2/sec*** ***dt*** | Correct equation for area of rectangle for u A = ***= (10 – x)× x2***  ***10***  | Correct chain of derivatives for r | Correct derivatives with correct solution for t |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply differentiation methods in solving problems.* | *Apply differentiation methods, using relational thinking, in solving problems.* | *Apply differentiation methods, using extended abstract thinking, in solving problems.*  |
| 3a | ***y = (x3 + 2)–4 /5*** ***yꞌ = – 4 (x3 + 2) -9/5 × 3x2*** ***5*** | Correct derivative.  |  |  |
| 3b | ***y = x2*** ***x – 2*** ***yꞌ = (x – 2)2x – x2 = 0 for stat pts*** ***(x – 2)2*** ***So 2x2 – 4x – x2 = 0*** ***x2 – 4x = 0*** ***x(x – 4) = 0*** ***x = 0 or 4***  | Correct derivative with correct solutions.  |  |  |
| 3c | ***y = xln(x) – 2x******yꞌ = x× 1 + lnx – 2***  ***x***  ***= 1 + lnx – 2*** ***= lnx – 1*** ***Decreasing if yꞌ < 0******So lnx – 1 < 0*** ***lnx < 1*** ***x < e1*** | Correct derivative for u | ***x < e*** ***is needed for r*** |  |

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| 3d |  ***M*** ***h*** ***θ*** ***Q 800 C******dh = - 90 h = tan θ******dt 800*** ***dh = 800sec2 θ = 800*** ***dθ cos2 θ******d θ = d θ × dh******dt dh d t******d θ = cos2 θ ×(–90)******dt 800******If h = 500 then tan θ = 5 so θ = 320*** ***8******d θ = cos2 θ ×–90 = cos232 ×–90******dt 800 800******d θ = –0.0809 rad/sec******dt*** |   ***dh = - 90***  ***dt*** ***h = tan θ*** ***800******and*** ***dh = 800sec2 θ dθ for u***  | all correct for raccept decreasing at 0.0809 rad/sec |  |
| 3e |  ***y***  A  S R ***y***   ***x 600*** **B P O Q C *x*** OC = 10 OA = 10√3 from ∆OAC ***Choose x,y axes as above.******Equ of AC is y = – √3 x + 10√3******AreaPQRS = 2x(– √3 x + 10√3)*** ***Area = – 2√3 x2 + 20√3 x*** ***Area = 2√3 (–x2 + 10x)*** ***d(Area) =2√3( – 2x + 10) = 0 for max*** ***dx******so x = 5, PQ = 10 and QR = 10√3 – 5√3*** ***= 5√3******max Area = 50√3*** | Use a coordinate system to approach the problem, use angles are 600 and calculate OA for u | Calculate correct equ for OA for r | Correct derivatives with correct solution. Finding max area for t |

**ASSESSMENT SCHEDULE**

####  **91579 Apply integration methods in solving problems**

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| **Achievement**  | **Achievement with Merit**  | **Achievement with Excellence**  |
|  *Apply integration methods in solving problems* involves*:*• selecting and using methods • demonstrating knowledge of concepts  and terms • communicating using appropriate  representations. |  *Apply integration methods, using relational thinking*, *in solving problems* involves one or more of: • selecting and carrying out a logical  sequence of steps • connecting different concepts or  representations • demonstrating understanding of  concepts • forming and using a model;and also relating findings to a context, or communicating thinking using appropriate mathematical statements. |  *Apply integration methods, using extended abstract thinking, in solving problems* involves one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or  proof • forming a generalisation;and also using correct mathematical statements, or communicating mathematical insight. |

Sufficiency for each question:

N0: No response, no relevant evidence.

N1: Attempt at ONE question showing limited knowledge of integration techniques.

N2: 1 **u**

A3: 2 **u**

A4: 3 **u**

M5: 1 **r**

M6: 2 **r**

E7: 1 **t** with minor errors ignored

E8: 1 **t** with full excellence criteria

**Judgement Statement**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Not Achieved** | **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| **Score range** | 0 - 6 | 7 - 13 | 14 - 18 | 19 -24 |

| Question ONE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.*  |
| 1a | ***= –4x – 3  – ln(x) + c*** ***3*** | Correct integration.  |  |  |
| 1b | === = 2 | Correct integration.  |  |  |
| 1c | ***v = 2t –*** Find the distance moved from ***t*** = 1 to ***t*** = 4***x = t2 + e 1 – t  + c******sub t = 4 x4 = 16 + e – 3  + c******sub t = 1 x1 = 1 + e0 + c = 2 + c******x4 – x1 = 14 + e – 3  ≈14.05 metres*** | Antiderivative correct. | Correct integration with correct solution.  |  |
| 1d |  ***dm =***  ***dt******t*** = 0 ***m = 8g*** and at ***t = 2*** hr, ***m = 1 g.*** = = ***kt + c******= kt + c******Sub t = 0, m = 8******6 = c******Sub t = 2, m = 1******3 = 2k + 6 so k = – 9******2 4*** ***= + 6******If m = 0 t = 24 = hours*** ***9*** | Correct diff equ with k and c found  = u | Correct time = r |  |
| 1e | The graphs have the equations ***y = (p2+1) – x2***and ***y = p2 where p > 1***  ***x2******Intersection when (p2***+ 1) ***– x2 = p2*** ***x2*** ***(p2 + 1)x2 – x4 = p2*** ***0 = x4 – (p2 + 1)x2 + p2*** ***(x2 – 1)(x2 – p2) = 0******x = ±1 and x = ± p******area =*** ***=*** ***–*** ***= 2p3 – 2p2 + 2p – 2***  ***3 3***  | Correct intersection for u | Correct idea of area = area under cubic – area under parabola (and vice versa) shown with equations for r.  | Correct solution showing correct integrations and correct mathematical logic and statements. |
|  |  |  |  |  |
| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
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| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| *f(x)* | 3 | 9 | 13 | 15 | 11 | 5 | 1 |
|  | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |

*A =2 3+ 1 + 4(9 + 15 +5) + 2(13+11)*  *3* *= 112* | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.*  |
| 2a | Correct calculation.  |  |  |
| 2b |  ***y* ꞌ = *4x3 + 3x2 – 4x + c sub x = 1, y* ꞌ = 4** ***4 = 4 + 3 – 4 + c so c = 1******y = x4 + x3 – 2x2 + x + d sub x = 1, y = 2*** ***2 = 1 + 1 – 2 + 1 + d so d = 1******y = x4 + x3 – 2x2 + x + 1***  | Correct integrations and constants evaluated and solution found = u.   |  |  |
| 2c |  A + B = ***dx = 8ln(8)***So A = 4ln(8)A = dx = 8ln(k)Solving 8ln(k) = 4 ln(8) ln(k) = ½ ln(8) = ln( 8  ½) k = 8 ½ | Correct value for total area for u. | Correct integration with correct value for k.  |  |
| 2d |  ***x =* 1**when ***y =* 1**, find ***y*** when ***x = 2*** = = ***ln(y) = – 3 + c***  ***x2*** sub ***x =* 1**and ***y =* 1*****0 = –3 + c so c = 3******ln(y) = – 3 + 3***  ***x2*** sub ***x = 2******ln(y) = – ¾ + 3 = 2 ¼*** ***y = e 2.25  ≈ 9.49***  | Correct integration with correct constant. | Correct integration with correct constant and ***y*** value found.  |  |

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| --- | --- | --- | --- | --- |
| Question TWO | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| 2e | =  =  = ***let u = ex + 1*** ***du = exdx*** ***=***  ***= ln(u) + c*** ***= ln( ex + 1) + c*** |  - | Correct manipulations of needed to integrate = r | Correct solution= t  |

| Question THREE | Evidence | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
|  |  | *Apply integration methods in solving problems.* | *Apply integration methods, using relational thinking, in solving problems.* | *Apply integration methods, using extended abstract thinking, in solving problems.*  |
| 3a | **= *dx******= x3 + 4x + 8ln*|*x*| *+ c*** ***3***  | Correct integration. **|*x*|** not essential |  |  |
| 3b |   **= =** =  | Correct integration.  |  |  |
| 3c |  ***x – 3 .***  ***x + 3 x2 + 0x + 4*** ***x2 + 3x*** ***– 3x + 4***  ***– 3x – 9***  ***+ 13*** = ***= x2 – 3x + 13 ln(x + 3) + c******2*** | Correct division or similar = u  | CorrectIntegration evaluated = r. |  |
| 3d | ***I*** = ***Let u = x + 2 so x = u – 2 and x – 2 = u – 4***  ***du = dx******I = equ* A** = = – 8= 4 | Uses substitution to obtain equ A | Evaluates integral correctly |  |
| 3e | ***dR = kR******dt******ln(R) = kt + c******sub t = 100, R = 150 :***  ***ln(150) = 100k + c equ1******sub t = 200, R = 90 :*** ***ln(90) = 200k + c equ2******subtracting :***  ***ln(90) – ln(150) = 100k*** ***so k = -0.005108*** ***sub in equ 1***  ***c = 5.5214******sub in the equ: ln(R) = kt + c*** ***ln(R) = –0.005108t + 5.5214******if R = 20 : ln(20) = –0.005108t + 5.5214*** ***so t ≈ 494 days***  | Obtains ***dR = kR*** ***dt***and states ***equ 1*** and ***equ 2*** for grade u | Obtains ***dR = kR*** ***dt***and calculates ***c***and ***k*** for grade r | Obtains***t ≈ 490 days*** with all correct mathematical statements for grade t.  |