**QUESTION ONE**

The shaded region in the diagram is bounded by the three curves:

y = 12 y

 x

y = 3x ( x – 3 )

 4

y2 = 18x

 A

 B

 C

Calculate this shaded area.

You may use a graphics

calculator but you must

explain every step of your

solution.

***To find A: y = 12 y2 = 18x***

 ***x***

***x = 12 so y2 = 18× 12***

 ***y y***

***y3 = 3.6.2.6 = 63***

***y = 6, x = 2***

***To find B: 12 = 3x(x – 3)***

 ***x 4***

***48 = 3x2(x – 3)***

***3x3 – 9x2 – 48 = 0***

***x = 4 on (GC)***

***Area between curves to the left of AC is:***

***½ x ½  – 3x2 + 9x dx = 10.5***

 ***4 4***

***Area between curves to the right of AC is:***

 ***dx = – 3x2 + 9x dx = 7.8***

 ***4 4***

 ***total area = 18.3***

**QUESTION TWO**

Show that the total area enclosed between the curves y = x(x + 2c)(x – 3c)

and y = – x(x + 2c)(x – c) is equal to 16c4. ( c > 0)

A

B

 - 2c c 2c 3c

*Find intersections: x(x + 2c)(x – 3c) = – x(x + 2c)(x – c)*

*So x(x + 2c)(x – 3c) + x(x + 2c)(x – c) = 0*

*Factorising x(x + 2c)* ***( x – 3c + x – c )***  *= 0*

 *x(x + 2c)* ***( 2x – 4c )***  *= 0*

 *x = 0 obviously, x = -2c obviously and* ***x = 2c***

*Area A = ∫ x(x + 2c)(x – 3c) + x(x + 2c)(x – c) dx*

 *= ∫ x3 – cx2 – 6c2x + x3 + cx2 – 2c2x dx*

 *0*

 *= ∫* ***2x3 – 8c2x dx***

 *- 2c 0*

 *=* ***x4*** ***– 4c2x2*** *= 0* ***–*** *8c4 – 16c4 = 8c4*

 *2 -2c*

 *2c 2c*

*Area B = ∫* ***– (2x3 – 8c2x) dx =*** ***4c2x2*** ***–******x4***

 *0 2 0*

 *= 16c4 – 8c4*

 *= 8c4*

*TOTAL AREA = 8c4 + 8c4 = 16c4*

**QUESTION THREE**

Find an expression, in terms of ***k***, for the area formed between the curves:

 ***y = x2 , y = x2 and y = 1***

 ***k x***

where ***k*** is a constant, and ***k*** > 1.

**Give the results of any integration needed to solve this problem.**

 ***y***

 ***y = x2***

 **P**

 ***y = x2***

 ***k***

 **Q**

 **A B**

 ***y = 1***

 ***x***

 **1 k*⅓ x***

***At P x2 = 1 so x3 = 1 and x = 1***

 ***x***

 1 1

so area A = **∫ *x2 – x2 dx = x3 – x3 = 1 – 1***

 ***k3 3k 3 3k***

 ***0***

***At Q x2 = 1 so x3 = k and x = k ⅓***

 ***k x***

 ***k ⅓  k ⅓***

so area B = **∫ *1 – x2 dx = ln (x) – x3***

 ***1  x k 3k***

 ***1***

 ***= ln(k ⅓) – 1 – ln (1) – 1 = ln(k ⅓) – 1 + 1***

 ***3 3k 3 3k***

***TOTAL area = A + B = ln(k ⅓)***