**THE IDEAL CAN.**

**(Volume V = πr2h Surface area S = 2πr2 + 2πrh)**

**Consider these cans which hold 400 mL = 400cm3**

***If r = 2 cm, h = 32 cm If r = 8 cm, h = 2 cm***

***V = π×22×32 ≈ 400 cm3 V = π×82×2 ≈ 400 cm3***

***S = 2π×22 +32×2π×2 = 136π cm2 S = 2π×82 +2×2π×8 = 160π cm2***

Obviously these two versions are not very “user friendly” because of their awkward sizes. **Notice, their volumes are equal but their surface areas are not.**

***The problem is, “What are the values of r and h so that the surface area S is as small as possible but the volume is still 400 cm3?”***

***r If the radius is r and the height is h***

***h then V = πr2h = 400***

***so that h = 400***

***πr2***

***The surface area S = 2πr2 + 2πrh***

***= 2πr2 + 2πr×400***

***πr2***

***= 2πr2 + 800 r – 1***

***dS = 4πr – 800r – 2  = 0 for min S***

***dr***

***4πr – 800 = 0***

***r2***

***4πr2 = 800***

***r2***

***r3 = 800***

***4π***

***So r = 3.993 and h = 400 = 7.986***

***π×3.9932***

***rounding sensibly:***

***r ≈ 4 cm and h = 400 ≈ 8 cm***

***π42***

**The ideal can which uses the least amount of metal**

**has the same HEIGHT as its DIAMETER!**

**Note: A lot of money could be saved by manufacturers because they could use less metal**

**to hold the same volume.**

**Extension: 1. Show that if the volume is V then for min S, *h = 2r***

**2. Show that for a container with no top the min S is when *r = h.***

***EXTENSION 1***

***If the volume to be used is V then:***

***V = πr2h so h = V***

***πr2***

***The surface area S = 2πr2 + 2πrh***

***= 2πr2 + 2πr×V***

***πr2***

***= 2πr2 + 2V r – 1***

***dS = 4πr – 2Vr – 2  = 0 for min S***

***dr***

***4πr – 2V = 0***

***r2***

***4πr = 2V***

***r2***

***r3 = V***

***2π***

***So r = V ⅓ and diameter = 2r = 2× V ⅓ = = V ⅓ ×2⅔***

***(2π)⅓  (2)⅓ (π)⅓ π⅓***

***and h = V***

***πr2***

***= V × (2π)⅔***

***π V⅔***

***= V ⅓ ×2⅔***

***π⅓***

**The ideal can which uses the least amount of metal**

**has the same HEIGHT as its DIAMETER!**

***EXTENSION 2***

***The surface area S = 1πr2 + 2πrh***

***= πr2 + 2πr×V***

***πr2***

***= πr2 + 2V r – 1***

***dS = 2πr – 2Vr – 2  = 0 for min S***

***dr***

***2πr – 2V = 0***

***r2***

***2πr = 2V***

***r2***

***r3 = V***

***π***

***So r = V ⅓***

***(π)⅓***

***and h = V***

***πr2***

***= V × (π)⅔ = V ⅓***

***π V⅔  (π)⅓***