**THE IDEAL CAN.**

**(Volume V = πr2h Surface area S = 2πr2 + 2πrh)**

**Consider these cans which hold 400 mL = 400cm3**

***If r = 2 cm, h = 32 cm If r = 8 cm, h = 2 cm***

***V = π×22×32 ≈ 400 cm3 V = π×82×2 ≈ 400 cm3***

***S = 2π×22 +32×2π×2 = 136π cm2 S = 2π×82 +2×2π×8 = 160π cm2***

Obviously these two versions are not very “user friendly” because of their awkward sizes. **Notice, their volumes are equal but their surface areas are not.**

***The problem is, “What are the values of r and h so that the surface area S is as small as possible but the volume is still 400 cm3?”***

 ***r If the radius is r and the height is h***

 ***h then V = πr2h = 400***

 ***so that h = 400***

 ***πr2***

***The surface area S = 2πr2 + 2πrh***

 ***= 2πr2 + 2πr×400***

 ***πr2***

 ***= 2πr2 + 800 r – 1***

***dS = 4πr – 800r – 2  = 0 for min S***

***dr***

 ***4πr – 800 = 0***

 ***r2***

 ***4πr2 = 800***

 ***r2***

 ***r3 = 800***

 ***4π***

 ***So r = 3.993 and h = 400 = 7.986***

 ***π×3.9932***

***rounding sensibly:***

 ***r ≈ 4 cm and h = 400 ≈ 8 cm***

 ***π42***

**The ideal can which uses the least amount of metal**

**has the same HEIGHT as its DIAMETER!**

**Note: A lot of money could be saved by manufacturers because they could use less metal**

 **to hold the same volume.**

**Extension: 1. Show that if the volume is V then for min S, *h = 2r***

 **2. Show that for a container with no top the min S is when *r = h.***

***EXTENSION 1***

***If the volume to be used is V then:***

***V = πr2h so h = V***

 ***πr2***

***The surface area S = 2πr2 + 2πrh***

 ***= 2πr2 + 2πr×V***

 ***πr2***

 ***= 2πr2 + 2V r – 1***

***dS = 4πr – 2Vr – 2  = 0 for min S***

***dr***

 ***4πr – 2V = 0***

 ***r2***

 ***4πr = 2V***

 ***r2***

 ***r3 = V***

 ***2π***

 ***So r = V ⅓ and diameter = 2r = 2× V ⅓ = = V ⅓ ×2⅔***

 ***(2π)⅓  (2)⅓ (π)⅓ π⅓***

***and h = V***

 ***πr2***

 ***= V × (2π)⅔***

 ***π V⅔***

 ***= V ⅓ ×2⅔***

 ***π⅓***

**The ideal can which uses the least amount of metal**

**has the same HEIGHT as its DIAMETER!**

***EXTENSION 2***

***The surface area S = 1πr2 + 2πrh***

 ***= πr2 + 2πr×V***

 ***πr2***

 ***= πr2 + 2V r – 1***

***dS = 2πr – 2Vr – 2  = 0 for min S***

***dr***

 ***2πr – 2V = 0***

 ***r2***

 ***2πr = 2V***

 ***r2***

 ***r3 = V***

 ***π***

 ***So r = V ⅓***

 ***(π)⅓***

***and h = V***

 ***πr2***

 ***= V × (π)⅔ = V ⅓***

 ***π V⅔  (π)⅓***