**PROBLEMS CONCERNING GETTING A LONG POLE AROUND A RIGHT ANGLED BEND.**

**1.**  P

 θ

 4 m L1

 θ

L2

Suppose this corridor has a right angled bend as shown.

The width of the corridor is 4 m

We wish to find the length of a pole PQ which can just be

moved around the corner (horizontally).

Q

 4 m

***4 = cosθ so L1 = 4***

***L1  cosθ***

***4 = sinθ so L2 = 4***

***L2  sinθ***

***The length of the pole L = L1 + L2 = 4 ( (sinθ) – 1  + (cosθ) – 1 )***

***dL = 4( (– 1) (sinθ) – 2 ( cosθ ) + (– 1)(cosθ) – 2 (–sinθ) )***

***dθ***

***dL = 4 ( sinθ – cosθ ) = 0 for max/min L***

***dθ cos2θ sin2θ***

 ***sinθ = cosθ***

 ***cos2θ sin2θ***

***so that sin3θ= cos3θ***

***and so tan3 θ = 1***

 ***tanθ = 1***

 ***θ = 450***

***L = L1 + L2 = 4 ( (sinθ) – 1  + (cosθ) – 1 )***

 ***= 4 ( √2 + √2 )***

 ***= 8√2 ≈ 11.31m***

The 1st derivative test is a little confusing!

|  |  |  |  |
| --- | --- | --- | --- |
| θ | <450 | 450 | >450 |
| dLdθ | negative | 0 | positive |

which implies a **minimum**!

But it really means that for angles θ > 450 a longer rod could have its end points on the walls AB and BC and be touching corner C (see fig 1.)

This rod cannot be moved any further round the corner towards C.

In fig 2, the rod is shorter and θ is decreasing but still > 450.

This rod also cannot be moved any further round the corner towards C.

**In fig 3, the angle θ = 450 and this is the shortest rod which touches both walls AB and BC and touches the corner point P.**

**(This is what the 1st derivative test is showing.)**

**It sounds contradictory but this is also the longest rod which can be moved around the corner!**

In fig 4, the angle θ is < 450 and the rod drawn is longer than that in fig 3.

This rod cannot be moved any further round the corner back towards A.

**1.** **2.**

A B A B

 P P

 **θ** **θ**

 C C

 **3. 4.**

 A B A B

 P P

 **θ** **θ**

**2.**  We will now repeat the theory for the following corridor of different widths.

 A B

 θ L1

 3m

 P

 θ L2

 ***2m C***

***3 = cosθ so L1 = 3***

***L1  cosθ***

***2 = sinθ so L2 = 2***

***L2  sinθ***

***The length of the pole L = L1 + L2 = 3(cosθ) – 1  + 2(sinθ) – 1 )***

***dL = (– 3)(cosθ) – 2 (–sinθ) ) + (– 2) (sinθ) – 2 ( cosθ )***

***dθ***

***dL = 3sinθ – 2cosθ = 0 for max/min L***

***dθ cos2θ sin2θ***

 ***3sinθ = 2cosθ***

 ***cos2θ sin2θ***

 ***sin3θ = 2 so tan3θ = 2***

 ***cos3θ 3 3***

***This means tanθ =*** $\frac{2^{\frac{1}{3}}}{3^{\frac{1}{3}}}$ ***so θ ≈ 410***

***We will find sinθ and cosθ from this triangle***

 ***h***

 ***h2 =*** $3^{\frac{2}{3 } }+2^{\frac{2}{3}}$$2^{\frac{1}{3}}$

***h =*** $(3^{\frac{2}{3 } }+2^{\frac{2}{3}} )^{\frac{1}{2}}$$3^{\frac{1}{3}}$

***sinθ =*** $2^{\frac{1}{3}}$ ***cosθ =*** $3^{\frac{1}{3}}$

 ***h h***

***The length of the pole L = 3(cosθ) – 1  + 2(sinθ) – 1***

 ***= 3h + 2h***

$3^{\frac{1}{3}}$$2^{\frac{1}{3}}$

 ***=*** $h( 3^{\frac{2}{3 }}+ 2^{\frac{2}{3}} )$

NB $b^{\frac{1}{2}} ×b^{1} $

 = $b^{\frac{3}{2}}$

 ***=*** $(3^{\frac{2}{3 } }+2^{\frac{2}{3}} )^{\frac{1}{2}}$ ***×*** $( 3^{\frac{2}{3 }}+ 2^{\frac{2}{3}} )$

 ***=*** $(3^{\frac{2}{3 } }+2^{\frac{2}{3}} )^{\frac{3}{2}}$

 ***≈ 7. 02 m***

Scale diagram:

 3cm 7cm

 410

 2cm

**3.** For the general case where one corridor is of width ***a*** and the other width ***b***

 we proceed as follows:

 A B

 θ L1

 ***a***

 P

 θ L2

 ***b*** C

***a = cosθ so L1 = a***

***L1  cosθ***

***b = sinθ so L2 = b***

***L2  sinθ***

***The length of the pole L = L1 + L2 = a(cosθ) – 1  + b(sinθ) – 1 )***

***dL = (– a)(cosθ) – 2 (–sinθ) ) + (– b) (sinθ) – 2 ( cosθ )***

***dθ***

***dL = asinθ – bcosθ = 0 for max/min L***

***dθ cos2θ sin2θ***

 ***asinθ = bcosθ***

 ***cos2θ sin2θ***

 ***sin3θ = b so tan3θ = b***

 ***cos3θ a a***

***This means tanθ =*** $\frac{b^{\frac{1}{3}}}{a^{\frac{1}{3}}}$

***We will find sinθ and cosθ from this triangle***

 ***h***

 ***h2 =*** $a^{\frac{2}{3 } }+b^{\frac{2}{3}}$$b^{\frac{1}{3}}$

***h =*** $(a^{\frac{2}{3 } }+b^{\frac{2}{3}} )^{\frac{1}{2}}$$a^{\frac{1}{3}}$

***sinθ =*** $b^{\frac{1}{3}}$ ***cosθ =*** $a^{\frac{1}{3}}$

 ***h h***

***The length of the pole L = a(cosθ) – 1  + b(sinθ) – 1***

 ***= ah + bh***

$a^{\frac{1}{3}}$$b^{\frac{1}{3}}$

 ***=*** $h( a^{\frac{2}{3 }}+ b^{\frac{2}{3}} )$

 ***=*** $(a^{\frac{2}{3 } }+b^{\frac{2}{3}} )^{\frac{1}{2}}$ ***×*** $( a^{\frac{2}{3 }}+ b^{\frac{2}{3}} )$

 ***=*** $(a^{\frac{2}{3 } }+b^{\frac{2}{3}} )^{\frac{3}{2}}$

***4. A slightly different problem is when a pole of fixed length is used and the***

 ***least width of one of the corridors is required.***

 ***A T B***

 ***θ 5 – h***

 ***2m***

 ***P***

 ***θ h***

 ***w V***

***We need to find the minimum width, w, of***

***the second corridor if the pole is 5m long.***

***Let PV = h so PT = 5 – h***

 ***C***

 ***2 = cosθ and w = sinθ***

***5 – h h***

***So 5 – h = 2 and h = w***

 ***cosθ sinθ***

***Adding we get 5 = 2 + w***

 ***cosθ sinθ***

***5 – 2 = w***

 ***cosθ sinθ***

***w = 5sinθ – 2sinθ***

 ***cosθ***

***w = 5 sinθ – 2tanθ***

***dw = 5cosθ – 2sec2θ = 0 for max/min w***

***dθ***

 ***5cosθ = 2***

 ***cos2θ***

 ***cos3θ = 2 so cosθ ≈ 0.7368 and θ ≈ 42.540***

 ***5***

***Subs in w = 5sinθ – 2tanθ***

 ***≈ 1.545 m***



Max point at (0.7425, 1.545) for ***x*** in rads

 Or ( 42.54, 1.545) for ***x*** in degrees

Again the 1st derivative test is a little confusing!

|  |  |  |  |
| --- | --- | --- | --- |
| ***θ*** | <42.540 | 42.540 | >42.540 |
| ***dw******dθ*** | positive  | 0 | negative  |

which implies a **maximum** value of w!

***What it really means is that when the pole is to be touching the wall AB at some point T and to be touching wall BC at some point V and to be touching the corner point P, the maximum width of the corridor would be 1.545m.***

***If it were any wider it could not touch at V.***

***(N.B. It could more easily fit round the corner if the 2nd corridor is >1.545m***

***but when it equals 1.545m, the pole touches the wall AB at T, the wall BC at V and the corner at P).***

 ***A T B***

 ***θ***

 ***2m***

 ***P***

 ***θ***

 ***V***

 ***w >1.545***

 ***C***